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Improving the Stability of a Morphodynamic Modeling System

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ABSTRACT

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Coastal area morphodynamic modeling systems couple modules for tidal hydrodynamics, wave propagation, sediment transport and bottom update. The non-linear coupling between these modules generates spurious oscillations and stability problems that are still poorly understood. This paper assesses and compares various methods to avoid the oscillations and improve the stability of a coastal area morphodynamic modeling system (MORSYS2D). The model is assessed with a simple case, the propagation of a sinusoidal bedform, and a complex natural system (Óbidos Lagoon, Portugal), whose exceptional dynamics enhances numerical problems in the morphodynamic models. The best results are obtained with a combination of methods: a predictor-corrector method to improve the implicitness of the solution, along-flux diffusion in the Exner equation to control oscillations, and a morphological factor below unity to reduce the Courant number. With these methods, the modeling system can be used with Courant numbers well above unity.

ADITIONAL INDEX WORDS: *Numerical oscillations, Óbidos lagoon*

INTRODUCTION

The need to predict the morphological evolution of surface water systems has fuelled the development of several types of models, using empirical, analytical, physical and numerical approaches. Numerical models are gaining popularity, as they encapsulate a growing understanding of the physical processes that govern sediment dynamics. Simultaneously, increasing computational resources allow century-long simulations (HIBMA et al., 2004).

Morphodynamic modeling systems are typically composed of different modules for tidal hydrodynamics, wave propagation, sediment transport and bottom updates. The strong non-linear coupling between these modules generates numerical problems that are still poorly understood. Even when all the modules are robust, their combination can lead to numerical oscillations and instabilities (JENSEN et al., 1999). These problems are typically addressed with numerical filters (JOHNSON and ZYSERMAN, 2002; CALLAGHAN et al., 2006) and the explicit introduction of diffusion in the bottom update equation (CAYOCCA, 2001). High-order time marching schemes are usually adopted to solve the bottom update equation (TANGUY et al., 1993), although they require additional mechanisms to dampen the oscillations. According to HUDSON *et al*. (2005), numerical oscillations in these systems stem from the separate resolution of the various equations. However, in engineering applications, their simultaneous solution is still incompatible with present computational power.

A morphodynamic modeling system under development for the past few years, MORSYS2D (FORTUNATO and OLIVEIRA, 2004), has been applied to several test cases, including both synthetic and real coastal systems, providing accurate and non-oscillatory results (FORTUNATO and OLIVEIRA, 2004; RAMOS et al., 2005). However, preliminary applications to a rapidly-evolving tidal inlet, the Óbidos Lagoon, highlighted the robustness limitations of the modeling system (OLIVEIRA et al., 2005). The present paper aims at presenting and assessing several strategies implemented in MORSYS2D to avoid the spurious oscillations.

THE MORPHODYNAMIC MODELING SYSTEM (MORSYS2D)

The modeling system, MORSYS2D, simulates the non-cohesive sediment dynamics in estuaries, tidal inlets and coastal regions, driven by tides, wind, river flows and waves (Figure 1). It integrates hydrodynamic models (ADCIRC, LUETTICH et al. 1991, and ELCIRC, ZHANG et al. 2004), a wave model (REF/DIFI, KIRBY and DALRYMPLE 1994) and a bottom update model (SAND2D, FORTUNATO and OLIVEIRA 2004). The system consists of a C-Shell script that runs independent models, manages the transfer of information between them and performs control checks.

The bottom update model simulates sand transport due to waves and currents using semi-empirical formulae and computes the resulting bed changes through the Exner equation (FORTUNATO and OLIVEIRA, 2004). This equation is solved with a node-centered finite volume technique based on an unstructured triangular grid

$$\Delta h^i = \frac{1}{1-\lambda}\nabla Q^i \qquad (1)$$

where Q is the sediment flux integrated over the morphological time step, λ is the porosity and h is depth relative to the mean water level. Sediment fluxes are computed at the center of the

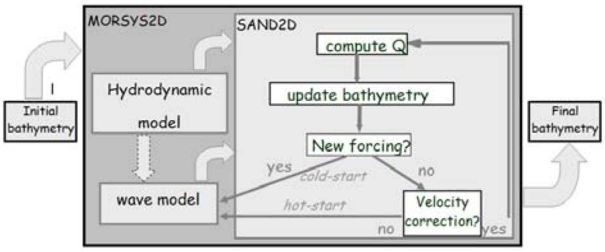

Figure 1. MORSYS2D solution procedure

triangles and integrated in time with a fourth order Runge-Kutta method, using an adaptive time step

$$h^{n+1} = h^n + \frac{1}{1-\lambda}\nabla \int_n^{n+1} q(u(t),\eta(t),h^n)dt \qquad (2)$$

where n is the time step, specified as an integer number of tidal cycles, u is the velocity and η is the tidal elevation. As the depth discretization is explicit, the stability of the solution should be limited by the Courant number. This number can be estimated as (ROELVINK, 2006)

$$Cu \approx \frac{bQ_s}{h\Delta x} \qquad (3)$$

where b is the velocity power in the transport formulae (typically between 3 and 5, depending on the specific formulation).

To mitigate the development of spurious oscillations, a non-linear filter is embedded in SAND2D. This filter operates when a local extreme appears in the depth and forces depth to a constant value at two adjacent nodes. The choice of the nodal depth is defined in a way that mass is conserved after the filtering process (FORTUNATO and OLIVEIRA, 2000). The non-linear filter has proved to be efficient at removing 2Δx oscillations, without introducing significant numerical diffusion in a transport model (OLIVEIRA and FORTUNATO, 2002). Henceforth, this formulation will be denoted as baseline formulation (BL).

METHODOLOGIES FOR OSCILLATION REMOVAL

Three different approaches were implemented to avoid spurious oscillations in morphodynamic simulations.

Predictor-Corrector Method

Since stability problems appear to stem from the explicit discretization of the Exner equation, a predictor-corrector scheme was implemented in the solution of this equation.

An estimate of depth at time n+1 is first calculated as

$$h^p = h^n + \frac{1}{1-\lambda}\nabla \int_n^{n+1} q(u(t),\eta(t),h^n)dt \qquad (4)$$

A fully or semi-implicit solution of this equation is then determined in the correction step as

$$h^{n+1} = h^n + \frac{1}{1-\lambda}\nabla \int_n^{n+1} q(u(t),\eta(t),h^*)dt \qquad (5)$$

where $h^* = \alpha h^p + (1-\alpha)h^n$

The implicitness of the method is determined by the parametre $\alpha \in [0,1]$. Equations (4) and (5) can be repeated iteratively, for a user-specified number of correction cycles.

Additional Diffusion Term in the Exner Equation

Several authors have included a diffusion-like term in the Exner equation to dampen numerical oscillations. Two different forms have been proposed for this term. CAYOCCA (2001) uses an isotropic diffusion term, where the diffusion coefficient is proportional to the magnitude of the sand fluxes. Other authors introduce diffusion only in the direction of the flow, by using a diffusive flux proportional to the advective flux in that direction (RAKHA and KAMPHUIS, 1997; ANTUNES do CARMO and SEABRA-SANTOS, 2002). The second approach is adopted here, as preliminary tests showed that cross-channel diffusion leads to the rapid accretion of the channels, thereby damping tidal propagation.

Since diffusion is added just for stability, a simple formula is adopted here, by replacing Q by Q_* in equation (1)

$$Q_* = Q + \varepsilon(1-\lambda)(|Q_x|\frac{\partial h}{\partial x}, |Q_y|\frac{\partial h}{\partial y}) \qquad (6)$$

where ε is a dimensionless diffusion coefficient. This new term was easily implemented in the baseline formulation and did not lead to a significant CPU time increase.

Morphological Factor

The large Courant numbers observed in the Óbidos Lagoon simulations suggest that reducing the time step could improve stability. However, this approach has some disadvantages.

First, velocities and water levels are fed into SAND2D in the frequency domain (i.e., through tidal amplitudes and phases, rather than time series). In order to compute the harmonic constants, a hydrodynamic simulation for at least a full tidal cycle is required for each morphological step, independently of the time step. Hence, reducing the morphological time step would increase the computational effort needed for the hydrodynamics.

Secondly, the main motivation for using morphological time steps that are an integer number of tidal cycles is to take advantage of the partial compensation of sediment fluxes on ebb and flood. This compensation results from the oscillatory nature of tides and leads to a residual sediment flux at least an order of magnitude smaller than the maximum instantaneous flux (FORTUNATO, 2007). Therefore, reducing the morphological time step by less than a factor of 10 could actually increase the Courant number.

As an alternative to reducing the time step, decreasing the Courant number was achieved through the morphological factor. Morphological factors are used to speed up simulations in slowly evolving systems (ROELVINK, 2006). Bathymetric changes are multiplied by a constant factor, f>1, after each time step. As a result, simulations for a period τ can reproduce the bathymetric changes for a period fτ. This approach changes the Courant number, which increases with f (ROELVINK, 2006)

$$Cu \approx \frac{fbQ_s}{h\Delta x} \qquad (7)$$

Here, advantage was taken of the Courant number dependence on f to reduce Cu. Since bathymetric changes in the Óbidos Lagoon are very rapid, using a morphological factor below unity appears appropriate. On the downside, this approach increases computational cost, since it requires repeating the simulation 1/f times. On the positive side, this approach leads to minor changes in the modeling system and allows for the use of the residual flux formulation of MORSYS2D.

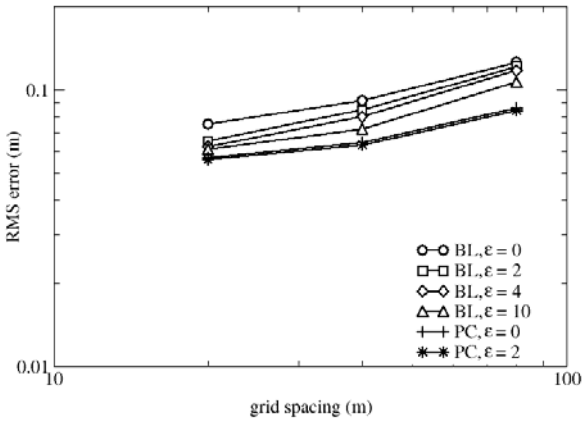

Figure 2. Exner dune test: convergence analysis.

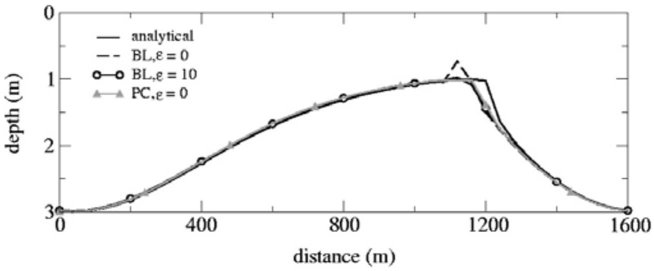

Figure 3. Exner dune test: comparison between numerical and analytical results for the 40 m grid.

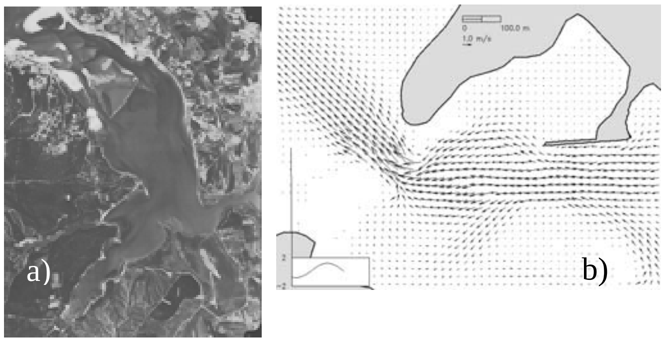

Figure 4. Óbidos lagoon: a) 2000 aerial photo; b) modeled ebb velocities at the inlet on an average tide.

RESULTS AND DISCUSSION

Test 1: Propagation of a Sinusoidal Sandwave

This test simulates the morphodynamic evolution of a dune under a stationary flow (KUDABKO et al., 2006). The water surface is assumed constant, and the sediment flux is set to q=(1-λ)/h. The analytical solution is given implicitly by

$$h(x,t) = h(x - c_z t, 0) \qquad c_z = h^{-2} \qquad (8)$$

where h is depth at time t and c_Z is the bed celerity. The initial configuration is given by

$$h(x,0) = 2 - \cos(2\pi x / L) \quad (9)$$

where L=1600 m is the dune wavelength.

A convergence analysis was done by running MORSYS2D for a range of time steps from 10 to 40 s and grid spacing from 20 to 80 m, keeping the maximum Courant number ($c_Z\Delta t/\Delta x$) fixed at 0.5. Simulations were performed with the baseline model (BL) for diffusion coefficients ε of 0, 1, 2, 4, and 10, and for the predictor-corrector (PC) with ε of 0 and 2 m^2/s. Since the Courant number is smaller than one, the factor f was set to 1 in this test.

Results show that the model is convergent as it approaches the analytical solution for increasingly smaller grid spacing (Figure 2). However, refining the grid increases the oscillations, which may be due to the reduction of the numerical diffusion inherent to the numerical approach. Including a diffusion coefficient on the order of unity eliminates the oscillations that purge the baseline formulation (Figure 3). However, the diffusion coefficient necessary to eliminate the oscillations depends on the grid-spacing.

The predictor-corrector method improves the accuracy (Figure 2) and reduces the diffusion coefficient necessary for non-oscillatory results (Figure 3).

Test 2: Morphodynamic Evolution of the Lower Óbidos Lagoon (Portugal)

Site description and simulations set-up

The Óbidos Lagoon is a small coastal system located on the western Portuguese coast (Figure 4a). The lower lagoon is characterized by a web of narrow channels and large sand banks, where velocities are very large (Figure 4b). The upper lagoon has small velocities and muddy bottom. A detailed description of the system is given in OLIVEIRA et al. (2006).

The morphological evolution of the lower lagoon is driven essentially by tidal currents and, to a smaller extent, by waves. The lower lagoon channels evolve rapidly, with displacements of the channels on the order of ten metres in a single tidal cycle. Aerial photos of the last decades show that the position and number of channels and sand banks is in permanent mutation (Figure 5). Bathymetric surveys and aerial photos show that meanders can form, grow and disappear in a few months (OLIVEIRA et al., 2006).

MORSYS2D simulations were conducted for 100 days, starting from the July 2001 bathymetry, after the repositioning and dredging of the mouth. These simulations were forced by tides alone, ignoring river flow inputs (which are negligible in the absence of floods) and wave action (which is mostly important during the maritime winter). Boundary conditions for the hydrodynamic model were taken from a regional tidal model (FORTUNATO et al., 2002). The morphodynamic simulations were conducted for a single sediment type (d50 = 0.6 mm), based on field data.

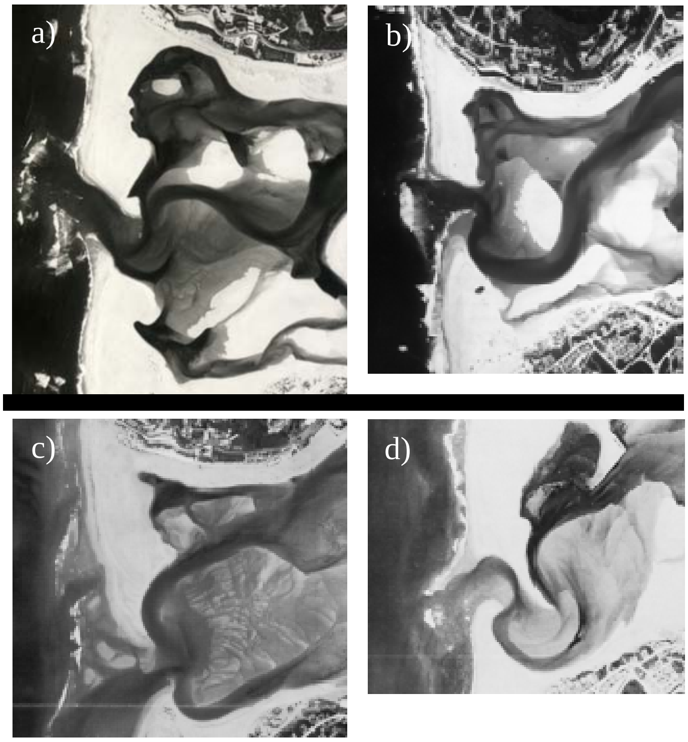

Figure 5. Aerial photos of the Óbidos lagoon: a) 1989; b) 1991; c) 1996; d) 2002.

Comparative assessment of the oscillation-removal strategies

The analysis was conducted by comparing the bathymetries from simulations using several combinations of the oscillation-removal strategies with those from the baseline formulation. All depths are presented in metres and relative to the mean sea level. Results for the baseline model (Figure 6a) illustrate the rapid development of numerical instabilities, after only a few time steps. The inlet mouth is the most critical area due to the large sediment fluxes and the small depths near the margins.

The large Courant numbers observed in the first time step of this morphodynamic simulation (Figure 6b) appear to be the source of the oscillatory behavior, since the oscillations start in the area with the largest Courant numbers (above 10).

Morphodynamic simulations in this system thus require additional strategies for model stabilization either through a reduction in the time step or the introduction of diffusion. However, given the magnitude of the observed Courant numbers, a reduction in the time step alone, to achieve *Cu* values below unity, can lead to prohibitive computational costs. A combination of strategies was thus exploited, based on the use of a morphological factor below unity, which was found necessary for stabilization, and other strategies.

Based on the results from the previous test and exploratory simulations, a diffusion coefficient of 4 was used for all simulations. A convergence analysis on the morphological factor was performed for values of *f* varying from 1 to 0.01, for two tidal cycles. Results show that the most of the domain converges for *f*=1, but the convergence near the mouth of the inlet only occurs for *f* around 0.05-0.2 (Figure 7), which requires a simulation of 5 to 20 tides to obtain a single tide. These results were confirmed in a 100 tides simulation.

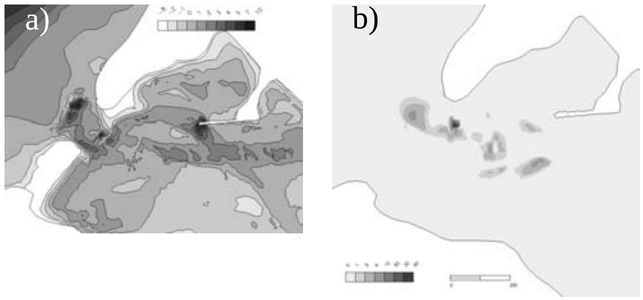

Figure 6. Baseline model: a) batimetry after 4 tidal cycles; b) Courant number in the first time step. For the evaluation of *Cu*, a minimum depth of 10 cm is specified.

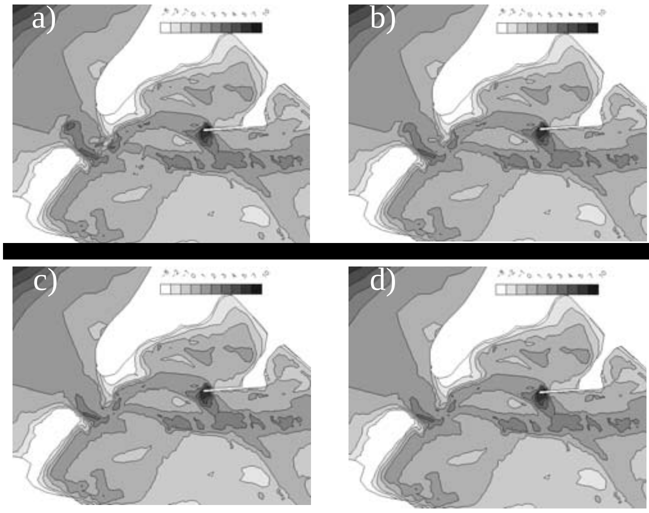

Figure 7. Morphological factor convergence analysis. Results after two tidal cycles: a) *f*=1.00; b) *f*=0.50; c) *f*=0.10 e d) *f*=0.05.

Since the use of the predictor-corrector in the dune test improved the accuracy and the stability, a simulation was conducted for this method, keeping ε=4 and *f*=0.1. Results (Figure 8) indicate that the predictor-corrector only contributes marginally to the stability of the results by reducing small oscillations.

Finally, one-year long simulations with *f*=0.2 are stable, even though the maximum Courant number varies between 1 and 8 during the simulation. Hence, provided some diffusion is introduced, the model is stable for Courant numbers well above unity.

CONCLUSIONS

Several strategies to avoid oscillations in morphodynamic models were presented and compared in a simplified 1D test and in a complex tidal inlet. These strategies include a predictor-corrector method, the introduction of diffusion in the Exner equation, and the use of a morphological factor below unity.

Results from the two applications suggest the following remarks:

- Diffusion can improve the results significantly. The best diffusion coefficient depends on the grid resolution.
- The predictor-corrector method can reduce the value of the diffusion coefficient required for stability in tests with sharp

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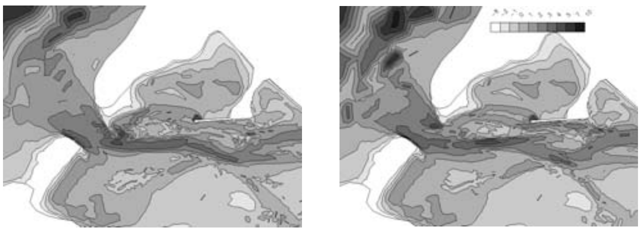

Figure 8. Predictor-corrector results after 365 tidal cycles for f=0.2: a) baseline; b) α=0.5 and 2 iterations

gradients. However, the advantages of its application were minor in the Óbidos lagoon.

- Using a morphological factor below unity appears as the primary solution for highly-dynamic systems with large Courant numbers. However, this approach increases excessively the computational time, which may compromise its application for long term simulations.
- Although the Courant number largely exceeds unity, results for the morphological factor of 0.2 are stable and physically realistic.

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