ON THE DEVELOPMENT OF A DISTORTED MODEL FOR A SEISMICALLY EXCITED LIQUID STORAGE TANK

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RESUMO

Current design codes of tanks under earthquake actions already include the consideration of tank-fluid interaction due to sloshing of the contained liquid. The advantage of large scale testing of tanks is not always possible, due to the enormous cost of a universal 3 or even 6 degree of freedom seismic table. Therefore, the purpose of the present work is to present the problem of conceptualising the small-scale testing of anchored tanks to unilateral seismic excitation, developing a needed know-how in Portuguese engineering laboratory infrastructure and also at FEUP. A scaled tank model, in accordance with the basic formulation of Vaschy-Buckingham Theorem ("Pi's" Theorem), cannot satisfy complete similarity. Then, a distorted model results, with a chain of distorted scales between the state variables of the seismically excited steel tank-fluid system. Choosing aluminium as the model material to scale the steel tank prototype, and water to scale the contained liquid, additional engineering conditions beyond dimensional analysis have to be considered, from which scaling factors for the state variables are determined.

1. INTRODUCTION

Surveys of recent moderate and severe earthquakes reveal that liquid storage tanks, and their appendices, have been extensively damaged. The need to insure integrity of this type of lifeline structures – usually containing water, wine, or hazardous chemicals – is essential for the minimisation of the disturbances to the quality of life of the earthquake affected societies. Design codes have been changing during last decades, upgrading the quantification of design loads and construction detailing in accordance with continuous evolution of theoretical analytical methodologies. Current design codes include already the consideration of tank-fluid interaction due to sloshing of the contained liquid. This interaction has been captured and modelled during tank tests in seismic tables of very few selected institutions, permitting to validate theoretical approaches but also to widen the understanding of the prototype real behaviour.

The advantages of prototype size testing are not possible at FEUP, due to the enormous cost of a universal 6 d.o.f. seismic table and its non-economically viable investment. Therefore, the purpose of this work is to present the problem of conceptualising the small-scale testing of anchored tanks to unilateral seismic excitation, developing a needed know-how in FEUP and Portugal laboratory infrastructure as well as constituting a calibration data-base on seismically excited tanks.

A scaled tank model, in accordance with the basic formulation of Buckingham Theorem ("Pi's" Theorem), cannot satisfy complete similarity. Then, a distorted model results, with a chain of distorted scales between the state variables of the seismically excited tank-fluid system. Choosing aluminium as the modal material to scale the steel tank prototype, and water to scale the contained liquid (water, wine, chemicals), additional conditions beyond dimensional analysis have to be considered, from which scaling factors for the state variables are determined [1].

2. GENERAL CONCEPTS AND MECHANICAL SIMILARITY

In a general mathematical sense, it is not possible to completely simulate the behaviour of a prototype (p) structural system (or any other) through model (m) tests. The complete similarity would have to be satisfied for the universe of (more and less) representative forces or causes present in the observed structure or phenomenon, which can never be accomplish among other reasons because of the discontinuous set of properties available for constructing the model. When through empirical and technical knowledge it is known that certain representative force dominates the prototype, or alternatively that certain dimensionless number (or "Pi" number) dominates, such value independent of scaling is used for deriving appropriate scaling laws.

That is the case of models satisfying partial (but first-order dominant) similarities of Froude, of Reynolds, of Euler, of Cauchy, of Strouhal, of Weber, and of the possible first-order dominant scales that can generally be defined through the so-called Vaschy-Buckingham "Pi" Theorem.

Difficulties arise when two or more representative forces are important or dominant in the prototype. Enforcing the validity of multiple similarities induces a difficult task for the modeller of well-designed and conducted model tests.

In what refers to the seismic analysis of bottom-supported tanks the available methodologies reveal a complex shell-fluid coupled interaction [1, 2, 3, 4, 5] that additionally indicates the dominance of multiple forces or causes present in the interaction phenomenon. Material time dependent non-linearity, like creep and temperature dependent behaviour, is considered negligible. The design of a model tank that simulates a prototype requires similitude relations, which can be derived through a dimensional analysis of the fluid-structure interaction describing the behaviour of the tank under an earthquake excitation.

In relation to the fluid-structure interaction, the most relevant mechanical variables or parameters of the bottom-supported metallic tank can be orderly classified as part of the following sources: geometry, container/contained materials, loading and response (Figure 1).

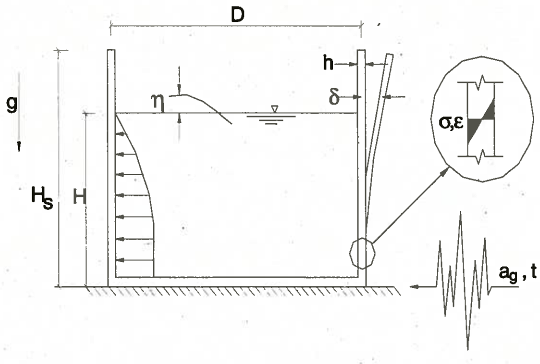

Fig 1: Model tank and some state variables

The tank geometric parameters considered, all with dimensional content of a linear dimension L, are: tank radius R, tank shell height H_t, tank shell thickness h, and fluid height in the tank H_f.

The tank or container material

parameters considered are: tank shell mass density ρ_t, with dimensional content ML^{-3}; elastic modulus E, with dimensional content $ML^{-1}T^{-2}$; dimensionless Poisson's ratio ν and percentage of critical damping ξ; and, generalised tank-fluid system angular frequency ω_t, with dimensional content T^{-1}.

The contained liquid material parameters are: mass density ρ_f, with dimensional content ML^{-3}; dynamic viscosity coefficient μ, with dimensional content $ML^{-1}T^{-1}$; compressibility modulus E_f, with dimensional content $ML^{-1}T^{-2}$; surface tensile stress δ, with dimensional content MT^{-2}; and, angular frequency of fluid sloshing oscillations ω_f, with dimensional content T^{-1}.

The tank-fluid system is located in a gravity field of gravitational acceleration g, with dimensional content LT^{-2}; is under a earthquake excitation described in the time domain t, with dimensional content T, by the horizontal ground acceleration $a_g(t)$, with dimensional content LT^{-2}.

The response parameters considered are: hydrodynamic pressure p, with dimensional content $ML^{-1}T^{-2}$; sloshing wave height η and tank shell radial displacements w, with dimensional content L; tank shell stresses σ, with dimensional content $ML^{-1}T^{-2}$; and, dimensionless tank strains ε.

3. DEVELOPMENT OF A GENERALISED MODEL

A generalised equation between these variables or parameters can be expressed [6, 7, 8, 9] in the form:

$$\begin{aligned}\phi(&R,H_t,h,H_f,\rho_t,E,\nu,\xi,\omega_t,\rho_f,\mu,\\ &E_f,\delta,\omega_f,g,a_g,t,p,\eta,w,\sigma,\varepsilon)=0\end{aligned} \quad (1)$$

Assuming that the function ϕ is dimensionally homogeneous, then selecting any $m=3$ parameters among the $n=22$ physical parameters and such that the corresponding dimensional matrix with respect to $\{L,M,T\}$ has characteristic equal to 3, permits to use such chosen variables as a new dimensional base of fundamental scientific variables or parameters.

But according to Vaschy-Buckingham 'Pi' Theorem [10,11,12] ϕ can alternatively be expressed by $\phi(\pi_1,\pi_2,\ldots\ldots,\pi_{n-m=19})=0$, in which the π_j *(j=1,2,...n-m=19)* are dimensionless combinations of the 3 fundamental variables and each of the remaining $n-m$ variables.

Therefore selecting $\{R,g,\rho_f\}$ as fundamental or base variables, after appropriate substitutions the generalised function ϕ of this coupled fluid-elasticity situation assumes the form:

$$\begin{aligned}&\phi(Fr,Eu,St,Re,Ca,We,\frac{h}{R},\frac{H_t}{R},\\ &\frac{H_f}{H_t},\frac{H_t}{R},\frac{\rho_t}{\rho_f},\frac{E}{\rho_f gR},\frac{a_g}{g},\frac{gt^2}{R},\\ &\frac{\eta}{H_t}\frac{H_t}{R},\frac{w}{R},\frac{\sigma}{\rho_f gR},\nu,\xi,\varepsilon)=0\end{aligned} \quad (2)$$

in which *Fr*, *Eu*, *St*, *Re*, *Ca* and *We*, represent the Froude, Euler, Strouhal, Reynolds, Cauchy and Weber 'Pi' numbers, and the remaining parameters are also dimensionless 'Pi' numbers.

Rigorous modelling of Froude and Reynolds similarities, involving gravity and viscous effects, is not possible. But since viscous forces are small in the phenomenon of tank liquid sloshing involving water, wine or crude oil and its chemical derivatives, the Reynolds number scale is ignored.

Also Cauchy similarity should be considered when the compressibility of the contained liquid cannot be neglected, which is definitely not the case of the above mentioned liquids behaving like incompressible fluids in small to medium liquid heights.

Finally Weber similarity is also omitted, because the model to be constructed is too large for the surface tensile stresses to be dominant.

With these simplifications the original model no longer will satisfy complete similarity but solely first-order similarity, and equation (2) is simplified to:

$$\phi(Fr, Eu, St, \frac{h}{R}, \frac{H_t}{R}, \frac{H_f}{H_t}\frac{H_t}{R}, \frac{\rho_t}{\rho_f}, \frac{E}{\rho_f gR}, \frac{a_g}{g}, \frac{gt^2}{R}, \frac{\eta}{H_t}\frac{H_t}{R}, \frac{w}{R}, \frac{\sigma}{\rho_f gR}, \nu, \xi, \varepsilon) = \tag{3}$$

$$\phi(\frac{R\omega_f^2}{g}, \frac{p}{\rho_f gR}, \frac{\omega_t}{\omega_f}, \frac{h}{R}, \frac{H_t}{R}, \frac{H_f}{H_t}\frac{H_t}{R}, \frac{\rho_t}{\rho_f}, \frac{E}{\rho_f gR}, \frac{a_g}{g}, \frac{gt^2}{R}, \frac{\eta}{H_t}\frac{H_t}{R}, \frac{w}{R}, \frac{\sigma}{\rho_f gR}, \nu, \xi, \varepsilon) \cong 0$$

in which the first term represents a modified form of Froude number, with the standard velocity square term substituted by the square product of a characteristic length and frequency (time inverse).

4. DISTORTED MODEL

The absence of Reynolds, Cauchy and Weber scaling laws does not result necessarily in a distorted model, since they were considered second-order effects. In fact, distortion between prototype and model occurs when one or more scaling laws of first-order dominant parameters is no longer valid, that is does not satisfy the unity (equality) criteria.

The small-scale tank model to be developed at a length scale λ_l (affecting R, H_t and H_f) will be conveniently designed in aluminium due to its material stress-strain constitutive law, corresponding to an elastic modulus scale λ_E. This choice allows for measurable deformations of the model tank. Had it be done in steel, like the prototype, the high rigidity of the small-scale cross-section would make it unusable.

The use of aluminium creates another source (although small) of material and structural performance distortion, related to the equivalent critical viscous damping ratio of the tank-fluid system.

However the scaling law of the Poisson's ratio parameter maintains the unity value $\lambda_\nu = 1$, which is an important factor to be kept in consideration for the small-scale modelling of tank shell performance.

The model incompressible fluid (assumed inviscid) is chosen as water, modelling the prototype fluids - water, wine or crude chemical derivatives - with a liquid mass density scale λ_ρ. If required, a higher mass density could be obtained by adding ferro-metallics or using magneto-hydro-rheologic fluids [13]. Then, another source of distortion between prototype and model is associated with the shell/fluid density ratio $\frac{\rho_t}{\rho_f}$, affecting dead load stresses and inertia forces.

The scaling law of gravity acceleration, and therefore of prototype-model ground acceleration, also maintains the unity value $\lambda_g = 1$.

Gravity forces are first-order dominant parameters that are properly scaled satisfying modified Froude number similarity. Therefore, imposing equality of modified Froude number between model and prototype, the following scaling law results: $\lambda_{\omega_t} = \lambda_t^{-1} = \lambda_l^{-1/2}$.

Notice that the free surface elevation is evaluated from the linearized dynamic free surface condition [14], without the negligible second order velocity square term, given by:

$$\eta = -\frac{1}{g}\frac{\partial \phi}{\partial t} - \underbrace{\frac{v^2}{2g}}_{\approx 0} \cong -\frac{1}{g}\frac{\partial \phi}{\partial t} \tag{4}$$

therefore

$$\begin{aligned}\lambda_\eta = \lambda_g^{-1}\left(\lambda_l\,\lambda_t^{-1}\right)^2 = 1\ x\ \lambda_l^2\,\lambda_\omega^2 = \\ = \lambda_l^2\,\lambda_l^{-1} = \lambda_l\end{aligned} \tag{5}$$

meaning that the free surface elevation is scaled by the geometric scale.

But with the above mentioned chosen scales $\lambda_l, \lambda_E, \lambda_\rho$ and $\lambda_g = \lambda_\nu = 1$, the small-scale tank model will generally be geometrically distorted, and will have a conditioned performance or distorted mechanical behaviour, with distinct scaling factors of the different mechanical parameters used. To obtain these scaling factors, through conditions beyond classical dimensional analysis, additional engineering relationships of the observed phenomena have to be used.

The 4th order differential equation, controlling the small forced vibrations of plates [15], states that:

$$(\rho_S)_t\,\ddot{w} + \frac{E_t\,h^3}{12(1-\nu^2)}\nabla^4 w = q(x,y,t) \tag{6}$$

in which $(\rho_S)_t$ and $q(x,y,t)$ are respectively the mass per unit area and the normal loading function.

After adequate substitutions from this last equation results the scaling law of tank thickness, controlling geometric distortion, expressed by: $\lambda_h = \lambda_\rho^{1/3}\ \lambda_l^{4/3}\ \lambda_E^{-1/3}$.

Pressure, either hydrostatic or hydrodynamic, is scaled by: $\lambda_p = \lambda_\rho\,\lambda_l$.

The scaling law of hydrostatic stresses, of type $\frac{pR}{h}$, is expressed by: $\lambda_\sigma^{(Hs)} = \lambda_p\,\lambda_l\,\lambda_h^{-1} = \lambda_\rho\,\lambda_l^2\,\lambda_h^{-1}$. The scaling law of hydrodynamic stresses, of the possible types $\frac{pR}{h}$ or $\frac{pR^2}{h^2}$ [16,17], assumes therefore values in the interval:

$\lambda_p\,\lambda_l\,\lambda_h^{-1} = \lambda_\rho\,\lambda_l^2\,\lambda_h^{-1} \le \lambda_\sigma^{(Hd)}$ and

$\lambda_\sigma^{(Hd)} \le \lambda_p\,\lambda_l^2\,\lambda_h^{-2} = \lambda_\rho\,\lambda_l^3\,\lambda_h^{-2}$.

The distorted model with a scaling law of tank thickness λ_h will deform also in a distorted manner, in which the shell radial deformation will be proportional to standard shell term pR^2/D. Therefore, the scaling law of tank predominant radial deformation is expressed by: $\lambda_w = \lambda_p\,\lambda_l^2\,\lambda_E^{-1}\,\lambda_h^{-1} = \lambda_\rho\,\lambda_l^3\,\lambda_E^{-1}\,\lambda_h^{-1}$.

Notice, however, that if the geometric scale λ_l (and also the liquid mass density scale λ_ρ) could be chosen in such a way that the derived pressure scale λ_p would equal the elastic modulus scale λ_E, then the thickness and the radial deformation scales would assume the same value of the geometric scale λ_l. In this case there would be no geometric distortion in the model, since all the primary and secondary variables or quantities with dimensional content of a linear dimension or length L would be equally scaled by λ_l. But the choice with such a coincidence has almost a null probability, due to infinite chances and discontinuous properties of available experimental materials, and therefore the reality of a distorted model cannot be disregarded.

5. COMPARISON TABLE OF SCALING LAWS

To assist the researcher and the experimentalist in choosing equipment and experimental methodologies, on the basis of technical interest for the seismic analysis of tanks and essentially on available budget, a comparison of various scaling laws is presented in Table 1.

These were obtained assuming that $\lambda_g = \lambda_v = 1$ and that the mentioned declared or chosen scales λ_ρ and λ_E are respectively $\lambda_\rho = \frac{\rho_m}{\rho_p} = \frac{1}{0.85}$ and $\lambda_E = \frac{E_m}{E_p} = \frac{700}{2100} = \frac{1}{3}$, corresponding to an aluminium tank filled with water but representing a distorted model of a prototype steel oil storage tank.

With these and with the additional selected length scale λ_l (the last one of the primary scales) result derived scales for the secondary variables, presented in tabular form for various values of λ_l.

Table 1 - Comparison of derived scaling laws, for various geometric length scales λ_l

Variable (scale)	$\lambda_l = 1/10$	$\lambda_l = 1/25$	$\lambda_l = 1/50$	$\lambda_l = 1/75$	$\lambda_l = 1/100$
Ground acceleration (λ_{a_g})	1	1	1	1	1
Frequency (λ_ω)	3.162	5.0	7.071	8.660	10.0
Time (λ_t)	0.316	0.2	0.141	0.115	0.10
Pressure (λ_p)	0.118	0.0470	0.0235	0.0157	0.0118
Tank thickness (λ_h)	0.0707	0.0208	0.00827	0.00481	0.00328
Hydrostatic stress ($\lambda_{\sigma Hs}$)	0.166	0.0905	0.0569	0.0435	0.0359
Hydrodynamic stress ($\lambda_{\sigma Hd}$)	0.166-0.235	0.0905-0.174	0.0569-0.138	0.0435-0.121	0.0359-0.109
Tank (radial) deformation (λ_w)	0.0499	0.0109	0.00341	0.00174	0.00108
Strains (λ_ε)	1	1	1	1	1

The scaling factors provided for the experimentalist in Table 1 give sufficient information to design a distorted model. From the previous table clearly depend range and sensitivities of equipment to be used in the experimental analysis of distorted model tanks, namely: LVDT's (or linear voltage displacement transducers), accelerometers, pressure transducers and strain gages. The available budget and the infrastructure space will dictate selections.

It is worth noting that besides selecting an earthquake exciter based on range, sensitivity and maximum displaced load (controlled by λ_l), the command equipment should also be able to scale down the acceleration-time and displacement-time records of the input seismic motion (controlled by the time scale above). The distorted model behaviour and performance, will permit to calculate response characteristics of a given prototype tank under predefined design assumptions.

6. CONCLUSIONS

On the basis of the Vaschy-Buckingham 'Pi' theorem of dimensional analysis, a generalised equation was derived for the experimental model analysis of seismically excited bottom supported tanks. Additionally, through considerations on standard equations of shell theories a distorted model was obtained and proposed. General scaling laws were additionally derived, permitting to the engineering modeller and researcher to choose equipment conditioned to available budget and infrastructure space.

KEYWORDS: Metallic tanks, Earthquake, Sloshing, Distorted model, Dimensional analysis.

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REFERENCES

[1] Barros, R.C., ***Preliminary Seismic Analysis and Design of Liquid Storage Tanks***, Numerical Techniques for Engineering Analysis and Design", Ed.: G.N. Pande & J. Middleton, ISBN 90-247-3564-5, NUMETA'87: Vol. 1, Section D - Engineering Analysis and Design, D-51, pp. 1-12, Martinus Nijhoff Publishers, Dordrecht, 1987.

[2] Haroun, M.A., Dynamic Analyses of Liquid Storage Tanks, Report to the National Science Foundation, EERL 80-04, California Institute of Technology, Pasadena - CA, 1980.

[3] Barros, R.C., ***Modelação da Resposta Sísmica de Tanques Circulares pelo M.E.F., incluindo Interacção Tanque-Líquido***, Proceedings VI Congresso Nacional de Mecânica Aplicada e Computacional, ISBN 972-8021-61-5, Vol. 2, pp. 1203-1212, APMTAC, Aveiro, 2000.

[4] Barros, R.C., ***Seismic Response Envelopes of a Tank Supported at the Base***, Computational Structures Technology, Section: Seismic and Earthquake Engineering, Ed.: B.H.V. Topping and Z. Bittnar, ISBN 0-948749-81-4 (CD-ROM, Paper 47, *20 pages*), ISBN 0-948749-82-2 (Extended Abstract, pp. 113-114), Civil-Comp Press, Stirling, Scotland, September 2002.

[5] Barros, R.C., ***Seismic Response of Tanks and Vibration Control of their Pipelines***, Journal Vibroengineering, Vol. 4, 2002 - No. 1 (8), Index 136, pp. 9-16, © Proceedings Research Center of the Public Institution Vibromechanika, ISSN 1392-8716, Vilnius, Lithuania, 2002.

[6] Murphy, G., Similitude in Engineering, The Ronald Press Company, NY, 1950.

[7] Langhaar, H.L., Dimensional Analysis and Theory of Models, John Wiley & Sons Inc., New York, 1951.

[8] Barros, R.C., ***On the Development of a Distorted Model for a Seismically Excited Liquid Storage Tank***, Oral presentation at the 1st International Materials Symposium, Materials 2001, Theme: Simulation and Modeling, SimMod 19, University of Coimbra, Portugal, April 2001.

[9] Barros, R.C., ***Desenvolvimento de Modelos Reduzidos para o Estudo Experimental da Fractura em Peças de Betão Estrutural***, Revista Mecânica Experimental, Edição da Associação Portuguesa de Análise Experimental de Tensões, ISSN-122 922, Nº 7, págs. 21-27, LNEC, Lisboa, 2002.

[10] Barbosa, J.N., Mecânica dos Fluidos e Hidráulica Geral, Vol. 1, Porto Editora, Porto, 1985.

[11] Carneiro, F.L., Análise Dimensional e Teoria da Semelhança e dos Modelos Físicos, Editora UFRJ, Rio de Janeiro, 1993.

[12] Harris, H.G. and Sabnis, G.M., Structural Modeling and Experimental Techniques, CRC Press, 2nd edition, 2000.

[13] Srinivasan, A.V. and McFarland, D.M., Smart Structures: Analysis and Design, Cambridge University Press, UK, 2001.

[14] Newmark, N.M. and Rosenblueth, E., Fundamentals of Earthquake Engineering, Prentice Hall Inc., Englewood Cliffs, New Jersey, 1971.

[15] Donnell, L.H., Beams, Plates and Shells, McGraw-Hill Book Company, New York, 1976.

[16] Flugge, W., Stresses in Shells, Springer-Verlag, 1st edition, Berlin, 1960.

[17] Harvey, J.F., Pressure Component Construction: Design and Materials Application, Van Nostrand Reinhold Company, New York, 1980.