DESIGN AND ANALYSIS OF A STRUCTURAL CONCRETE MINI-FRAME

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ABSTRACT

In the present work is presented the design and analysis of a structural concrete mini-frame reinforced with steel fibres. The objective is to optimize the mini-frame in order to support the maximum impact load with restrictions in term of the maximum and minimum dimensions and in the steel weight and global weight. The analysis of the mini-frame was made by a simplified linear analysis, by linear elastic Finite Element Method and a non-linear Finite Element Method. The methods give similar results in the elastic branch and the non-linear analysis defines the ultimate static load.

1 - INTRODUCTION

The present work describes the design and analysis of a structural concrete mini-frame, reinforced with steel fibres that won the first prize in a competition at the Conference Betão Estrutural 2000 hosted by the Faculty of Engineering of Porto, Portugal.

The objective of the contest was to design a mini-frame supporting the maximum impact load acting at the middle span, having prescribed maximum dimensions and weight. The materials used in the mini-frame, layout of the reinforcement and methods of analysis are described in the present work.

The concrete is mixed with steel fibres and also reinforced with steel bars.

Three methods of analysis were used after the design of the mini-frame. The first is a simplified linear analysis, the second is a linear elastic analysis made by the Finite Element Method for the analysis of structures with two dimensions and the last is a nonlinear analysis by the Finite Element Method also.

2 - DESCRIPTION OF THE MINI-FRAME

The mini-frame is a reinforced concrete structure with a maximum length of 650mm, a maximum height of 400mm and a minimum of 300mm at the span.

The composition of the concrete used in the mini-frame satisfies the following expression:

$$1 = m + c + A + Vv \qquad (1)$$

where m represents the volume of aggregate per volume of concrete, c the volume of cement per volume of concrete, A the volume of water per volume of concrete and Vv is the volume of voids per volume of concrete. The quantity of aggregate in the previous expression m was obtained from Faury's law. The strength of the concrete was defined by testing 6 cubes with 150X150X150 mm^3, as is shown in Figure 1. In order to improve the quality of the concrete in terms of the water-cement ratio, workability and other properties, the additives termed Sikament FF, Sika-rapid 1 and Sikacrete HD were used. The water/ cement ratio was fixed and equal to 0.4, as is indicated in Table 1. The quantities of all the materials used in fabricate the concrete are resumed in Table 1. The mean strength of the concrete obtained experimentally in cube was equal to 90 MPa and corresponds to a mean strength in cylinder f_{cm} equal to 80 MPa.

In Figure 2 are plotted the load displacement diagrams obtained in the compression tests of the cubes performed. In Table 2 are resumed the main properties and the results of these tests.

The reinforcement of the mini-frame was made with steel bars and steel fibres. The yield strength of the steel is equal to 160 MPa. The stress-strain diagram of steel is represented in Figure 3. Figure 4 represents the mini-frame.

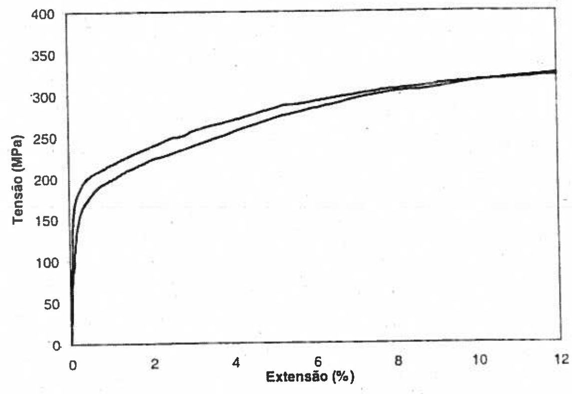

Figure 3 - Stress-stain diagram of steel

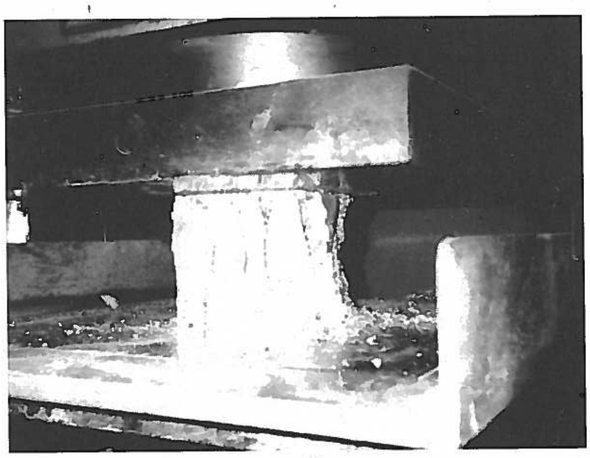

Figure 1 – Cube in testing

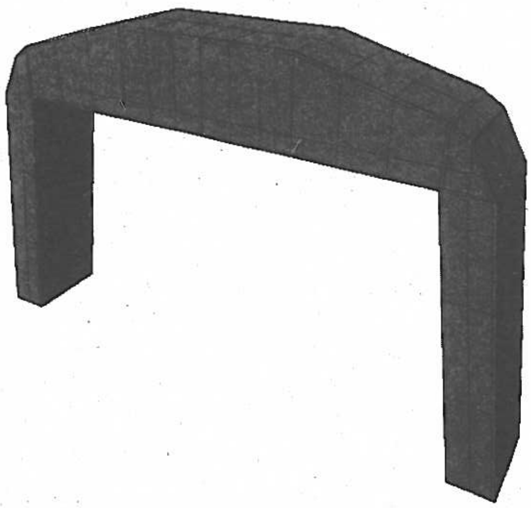

Figure 4 - Geometry of the mini- frame

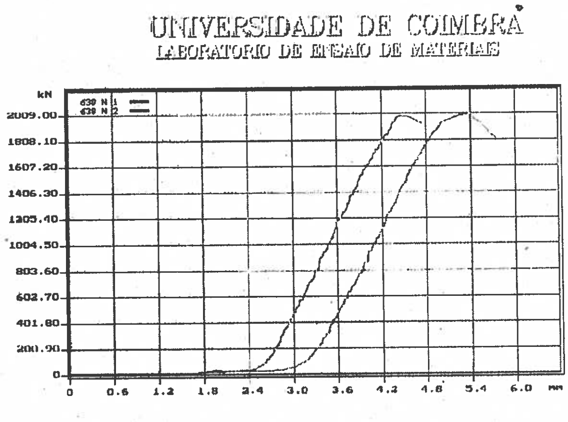

Figure 2 - Load displacement diagrams obtained

3 - MODELS IN THE DESIGN OF THE MINI-FRAME

Due to the contest regulations regarding the reinforcement steel that should be used (including steel bars and steel fibres) and due to the restriction of the maximum weight of the mini-frame, the design of the mini-frame took into account the following considerations:

1 - Different types of concrete were made in order to optimise the mixtures with the objective of maximise the compressive strength of the concrete. The specific weight of the optimum concrete was measured and used in the design.

Table 1 – Composition of the concrete

	Cement (kg)	Sand (kg)	Aggregate (kg)	Steel Fibres (kg)	Fume Silica (kg)	Hardening Accelerator (l)	Superplasticizers (l)	Water w/c
/m^3	500	480	1070	30	30	2	12	0.4

Table 2 – Results of the tests

	Weight (kg)	Ultimate Load (KN)	Max. Deformation (mm)	Ultimate Strain (%)	Ultimate Stress (MPa)	Density (g/cm^3)
Concrete	8.00	2008.58	5.73	-98.08	89.27	2.46

2 - The shape of the mini- frame was defined in order to maximise its resistance and stiffness. Since the critical section is at the middle span of the frame, the maximum admissible area was used at that zone.

3 - In order to reduce the global weight the dimension of the columns were reduced to a minimum value, although instability was taken into account to avoid the columns collapse.

4 - The design of the reinforcement was made according the weight limitation but following the necessities in terms of bending moment and transversal load. The ratio of fibre weight per unit volume of concrete used in the composition was equal to 30Kg/ m^3 (weight per volume of concrete), corresponding to a total of 0.608 Kg of fibres in the mini-frame. Since the global weight of steel is limited to 2.000Kg, only 1.392 Kg of reinforcing steel bars could be used. The layout of the reinforcement was made in order to support the different forces with a minimum length and is represented in Figure 8.

5 - The effects of the cyclic nature of the load or fatigue were not considered.

6 – The mini-frame was submitted to a preliminary design by a linear analysis, although due to the dimensions it can not be considered a linear element.

7 - The mini- frame was considered doubly supported at the end of both columns.

8 - Since the critical section is at the middle span the ultimate load of the mini-frame is calculated using the ultimate bending moment of this section.

9 - The dynamic analyses were made following energetic considerations.

4 - METHODS OF ANALYSIS

4.1 Simplified analysis

Following a linear analysis it is concluded that the maximum bending moment in the mini-frame is at the middle span of the beam.

The calculation of the ultimate bending moment is made considering that rupture occurs in the reinforcement for a strain equal to 10‰ that is called rupture by the steel. The deformation of the section is represented in Figure 5a).

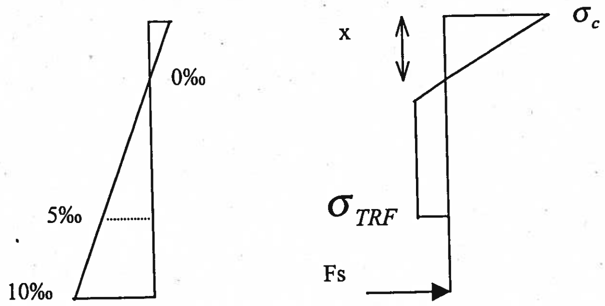

Figure 5.a) Deformation of the section at rupture
b) Stress at the section

In Figure 5a) is also shown the deformation of the fibres at rupture that is taken equal to 5‰ and the corresponding stress σ_{TRF}. The stress σ_{TRF} is the fibres tension in the cracked sections indicated in Bernhard (1995), given by

$$\sigma_{TRF} = \left[K_3 \times \frac{lf}{df} + K_4 \cdot \right] Vf \times \delta_{rT} \qquad (2)$$

where K3, K4 are constants depending on the concrete and the fibres used, V_f is the volume of fibres per unit volume of concrete, lf is the fibres length, df is the diameter of the fibres, δ_{rT} is the ultimate compression stress of the concrete matrix. The value σ_{TRF} obtained for this case is equal to 1.19 MPa.

The stress in concrete reinforced with fibres is defined by a linear diagram in compression and a constant value in tension up to the stain of 5‰, as represented in Figure 5b). The ultimate bending moment obtained for the middle span section is equal to 1.4838KNm, and the corresponding ultimate static load is equal to 16.34 KN.

The dynamic analysis made consists of computing the elastic potential energy U_e that is

$$Ue = \frac{1}{2} \times P' \times \delta = 1.07 \times 10^{-3} \qquad (3)$$

where δ is the displacement due to a static loading of 16.34 KN, that is $\delta = 1.307 \times 10^{-4}$m and P´=16.34 KN.

Computing the dynamic coefficient ψ_1 which is determined by the following expression:

$$\psi_1 = \frac{P'}{P} = 1 + \sqrt{1 + \frac{U_c}{U_e}} \qquad (4)$$

where P is the dynamic load. Taking h=3.0 m the result of P obtained is 45.69Kg.

4.2. Linear elastic analysis by the Finite Element Method

A linear elastic analysis was made using a two dimensional Finite Element Code. The stresses obtained are represented in Figure 6 and the corresponding displacements in Figure 7. In Figure 6 is also represented the mesh used in the analysis. In Figure 8 is represented the layout of the reinforcement.

The displacement in the mid-span obtained from the elastic analysis made with the F.E.M was $\delta = 1.283 \times 10^{-4}$m.

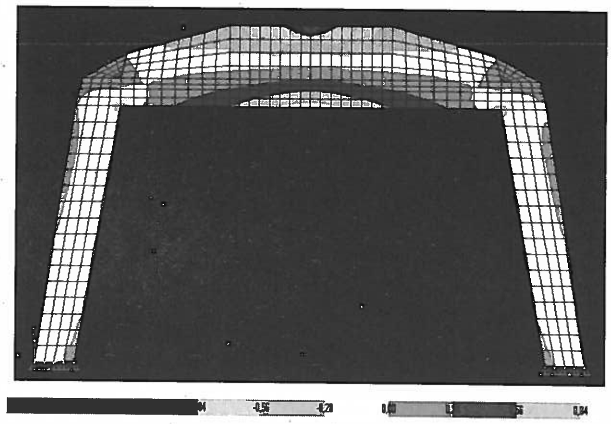

Figure 6 - Stresses in the mini-frame

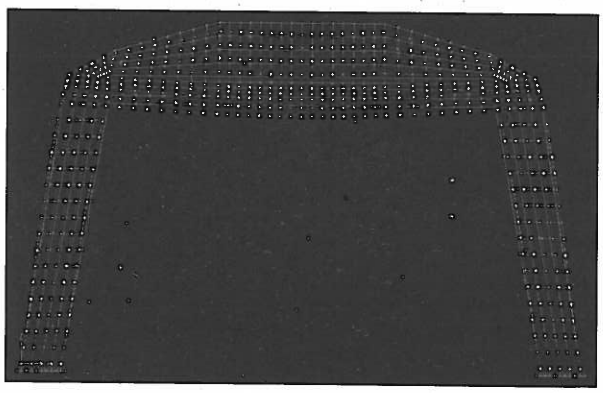

Figure 7 - Displacements in the mini- frame

4.3 Non-Linear analysis by the Finite Element Method

A non-linear analysis with the finite element method was also made. The displacement obtained for the load P applied at the middle span of 16.34 KN was

$$\delta = 1.245 \times 10^{-4}\text{m}$$

Increasing the load P after this value the plastic behaviour of the mini-frame was observed. According to this analysis the frame collapses with the following load and displacement at the middle span

$$P=29.4KN \quad \text{and} \quad \delta=3.8\times10^{-4}m$$

In this analysis a coarse mesh was used, while in the linear analysis the mesh was fine. This fact did not seem to influence the results, since in the elastic branch they are very similar, as can be concluded from the displacements obtained in the two analyses.

The tension for crack opening in the concrete with fibres was defined according to Bernhard (1995). For the present concrete σ_{TR} is equal to 5.12 MPa.

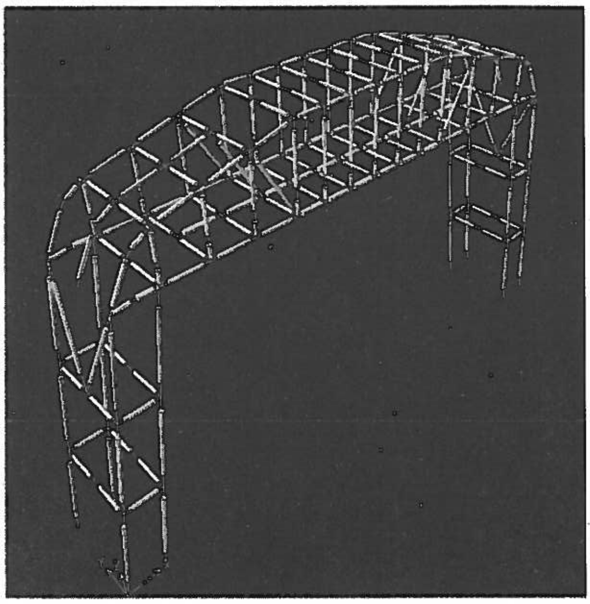

Figure 8 - Layout of the reinforcement

5 - COMPARISION OF RESULTS

The results obtained in the different analysis are represented in Figure 9. It can be concluded that in the elastic branch the three analysis are coincident and that the nonlinear analysis presents a non-linearity after a load equal to 15KN and when it approximates 30KN the collapse occurs by yielding of the reinforcement.

6 - CONCLUSIONS

All the methods gave nearly the same values in the elastic branch. In the diagram is visible the reduction of stiffness of the structure after the crack formation at the middle-span section.

The real testing provided a smaller value of the collapse load. This was due to

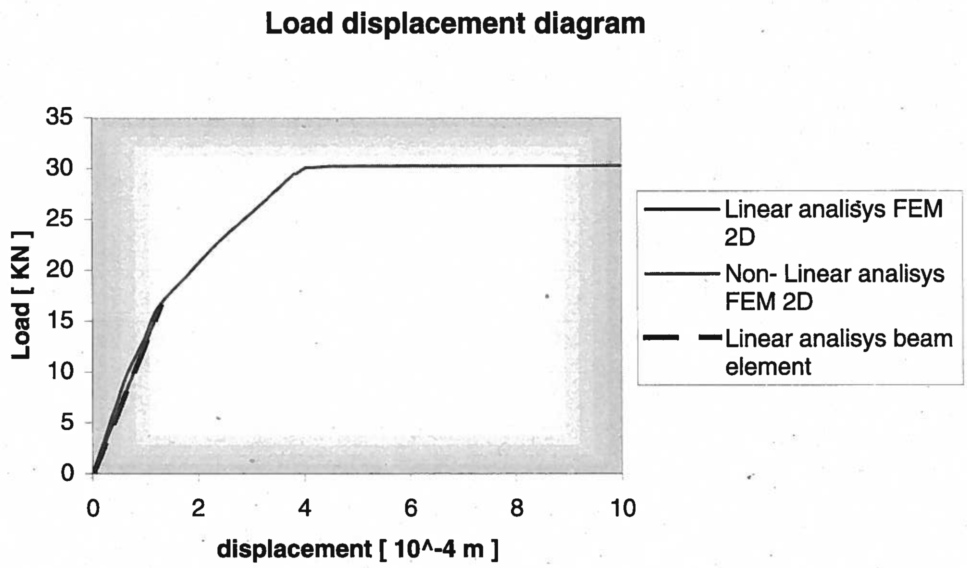

Figure 9.- Load-displacement diagrams

different conditions. The applied load was dynamic and with a strong cyclic component (over 20 impacts). The failure criterion was the damage of an egg placed bellow the mini-frame which occurred for a displacement of approximately 5 cm. The present mini-frame supported the maximum impact load in the contest but unfortunately no measures for the definition of the global load-displacement diagram were taken during the testing.

ACKNOWLEDGMENT

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