EXPERIMENTAL VERIFICATION OF MATHEMATICAL MODEL FOR ORIENTED DAMAGE OF CONCRETE

Bogucka*,J. ; Debinski**, J.; Litewka***, A.; Mesquita****, A.B.

* Associate Professor, Poznan University of Technology, Poznan, Poland
** Assistant, Poznan University of Technology, Poznan, Poland
*** Full Professor, Universidade da Beira Interior, Covilhã, Portugal
**** Assistant, Universidade da Beira Interior, Covilhã, Portugal

   

Resumo

Verificação experimental do modelo matemático sobre os danos orientados no betão

Para a verificação da hipótese relacionada com a anisotropia de um betão, induzida da deformação e dos danos, foram realizados testes não padronizados de deformabilidade do betão de modo a medir a alteração das suas constantes de elasticidade. Constatou-se que a carga aplicada uniaxialmente sobre provetes de betão B20, originou uma monotropia do material, que é descrita através de cinco constantes de elasticidade. Os resultados obtidos permitem-nos corroborar a validade da teoria apresentada, e formular com mais rigor o modelo da equação da evolução dos danos que é a relação básica deste modelo teórico.

1. INTRODUCTION

One of the factors, which decide on fulfilment of rigorous requirements of security and integrity of structures, formulated in design codes, is appropriate knowledge of mechanical properties of structural materials. This applies not only to new materials but also to those traditionally used, like concrete being in common use in dams, bridges, nuclear and conventional power stations. Such a deeper study of the material behaviour includes more realistic theoretical description of the material response as well as experiments suitably oriented to analysis of its properties.

Existing theoretical models formulated to describe a behaviour of a concrete subjected to multiaxial state of stress (Geniev et al., 1974 and Chen, 1982) are based on the analysis of plastic properties of this material. However, one can find there some suggestions that the deformability of the concrete is governed by development of oriented cracks referred to as anisotropic damage which grows in the loading process. Some analyses of this problem done by employing the methods of continuous damage mechanics are presented in earlier papers (Eimer, 1971 and Dragon, 1976). The first attempt to measure experimentally the damage induced anisotropy of a concrete is due to Mitrofanov and Dovzenko (1991) but stress-strain relations for this material formulated with account for its anisotropy are presented by Karpenko (1987) and Baikov (1990).

Some of the authors of this note formulated in their earlier papers (Litewka et al.,1996, and Litewka and Debinski, 1997) a mathematical model capable to describe non-linear stress-strain relations for a concrete subjected to multiaxial state of stress. The relevant constitutive equations were derived by employing the methods of continuous damage mechanics and the theory of tensor function representations. The aim of this paper is to verify experimentally the validity of that theoretical model as well as to detect the existence of the deformation and damage induced anisotropy of a concrete. To this end the non-standard tests of concrete deformability were performed in order to measure directly a modification of its constants of elasticity and to identify the unknown coefficients that appears in the equations of the mathematical model proposed.

2. THEORETICAL MODEL

Mathematical model for deformability of the concrete, to be verified in this paper, is based on continuous damage mechanics and in particular on the assumption on tensorial nature of the material damage (Murakami and Ohno, 1981, Betten, 1983 and Litewka, 1991). The explicit form of the relevant constituve equations was found by employing the theory of tensor function representations (Spencer, 1971 and Boehler, 1987). Practical application of the above theories to describe a non-linear behaviour of a concrete was explained elsewhere (Litewka et al.,1996, and Litewka and Debinski,1997) and that is why the final form of the relevant equations only will we shown here.

First group of the constitutive equations consists of the damage evolution equation

$$\Omega_{ij} = A s_{kl} s_{kl} \delta_{ij} + B\sqrt{\sigma_{kl}\sigma_{kl}}\,\sigma_{ij} \quad , \qquad (1)$$

where Ω_{ij} is a second order damage tensor responsible for a current state of a deteriorated material structure, whereas δ_{ij} is a Kronecker delta and σ_{ij}, s_{ij} stand for the stress tensor and stress deviator, respectively. Two scalar parameters A and B are unknown material constants to be determined experimentally.

The second group of equations consist of the stress strain relations

$$\begin{aligned}\varepsilon_{ij} &= -\frac{\nu_0}{E_0}\delta_{ij}\sigma_{kk} + \frac{1+\nu_0}{E_0}\sigma_{ij} + \\ &+ C(\delta_{ij}D_{kl}\sigma_{kl} + D_{ij}\sigma_{kk}) + \\ &+ 2D(\sigma_{ik}D_{kj} + D_{ik}\sigma_{kj})\end{aligned} \qquad (2)$$

where ε_{ij} is the strain tensor, and the equation

$$\begin{aligned}A_{ijkl} &= -\frac{\nu_0}{E_0}\delta_{ij}\delta_{kl} + \\ &+ \frac{1+\nu_0}{2E_0}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \\ &+ C(\delta_{ij}D_{kl} + D_{ij}\delta_{kl}) + \\ &+ D(\delta_{ik}D_{jl} + \delta_{jl}D_{ik} + \delta_{il}D_{jk} + \delta_{jk}D_{il})\end{aligned} \qquad (3)$$

which defines the material constants of orthotropically damaged solid. Equations (2) and (3) contain the Young modulus E_o and Poisson ratio ν_0 for original undamaged material, two unknown constants C and D to be determined experimentally and the second order symmetric damage effect tensor D_{ij}. The principal values D_1, D_2 and D_3 of the damage effect tensor D_{ij} are related to the principal components Ω_1, Ω_2 and Ω_3 of the damage tensor Ω_{ij} as follows (Litewka,1991)

$$D_i = \frac{\Omega_i}{1-\Omega_i} \quad , \quad i = 1,2,3 \, . \qquad (4)$$

To enable the comparison of theoretical predictions with experimental results obtained for uniaxial compression, the equations (1) and (2) were specified for this state of stress determined by the minimum principal stress σ_3. Taking into account equation (4), the following non-linear stress strain relations were obtained:

$$\varepsilon_1 = \varepsilon_2 = -\frac{\nu_0}{E_0}\sigma_3 + $$

$$+ C\left[\frac{2A\sigma_3^{\,3} - 3B\sigma_3^{\,3}}{3 - 2A\sigma_3^{\,2} + 3B\sigma_3^{\,2}} + \frac{2A\sigma_3^{\,3}}{3 - 2A\sigma_3^{\,2}}\right] \quad (5)$$

$$\varepsilon_3 = \frac{\sigma_3}{E_0} + (2C + 4D)\frac{2A\sigma_3^{\,3} - 3B\sigma_3^{\,3}}{3 - 2A\sigma_3^{\,2} + 3B\sigma_3^{\,2}} \quad (6)$$

where ε_3 is longitudinal principal strain and ε_1, ε_2 are transverse principal strains.

Equation (3) represents nine material constants of orthotropy of a concrete subjected to multiaxial state of stress. In the specific case of uniaxial compression, a deformation induced oriented damage of the material results in development of a transverse isotropy with the plane of isotropy perpendicular to the direction of the compressive load. Five constants which describe the transverse isotropy can also be determined from equation (3) which in the case of the uniaxial compression takes a form of five following relations

$$\frac{1}{E} = A_{1111} = \frac{1}{E_0} + (4C + 8D)\frac{A\sigma_3^{\,2}}{3 - 2A\sigma_3^{\,2}} \quad (7)$$

$$\frac{1}{E'} = A_{3333} = \frac{1}{E_0} + $$

$$+ (2C + 4D)\frac{2A\sigma_3^{\,2} - 3B\sigma_3^{\,2}}{3 - 2A\sigma_3^{\,2} + 3B\sigma_3^{\,2}} \quad (8)$$

$$\nu' = A_{1133}E' = \frac{E'}{E_0}\nu_0 - $$

$$- CE'\left(\frac{2A\sigma_3^{\,2}}{3 - 2A\sigma_3^{\,2}} + \frac{2A\sigma_3^{\,2} - 3B\sigma_3^{\,2}}{3 - 2A\sigma_3^{\,2} + 3B\sigma_3^{\,2}}\right) \quad (9)$$

$$\nu = A_{1122}E = \frac{E}{E_0}\nu_0 - 4CE\frac{A\sigma_3^{\,2}}{3 - 2A\sigma_3^{\,2}} \quad (10)$$

$$\frac{1}{G'} = A_{1313} = \frac{2(1 + \nu_0)}{E_0} + \quad (11)$$

$$+ 4D\left(\frac{2A\sigma_3^{\,2}}{3 - 2A\sigma_3^{\,2}} + \frac{2A\sigma_3 - 3B\sigma_3^{\,2}}{3 - 2A\sigma_3^{\,2} + 3B\sigma_3^{\,2}}\right)$$

The equations (7), (8), (9), (10) and (11) represent the non-linear relations between a compressive stress σ_3 applied to a material and an actual value of each constant of anisotropy. The material in its initial state, where $\sigma_3 = 0$, is isotropic and its properties are described by two constants of elasticity E_0 and ν_0.

3. EXPERIMENTS

The purpose of the tests performed is to detect experimentally the existence of the material anisotropy that develops in the loading process due to internal damage growth. To this end the modification of the magnitudes of the constants E and E' was studied experimentally by means of electric strain gauge measurement combined with ultrasonic pulse velocity technique. The analysis of the coefficients of lateral deformation ν and ν' was also attempted, however in this case the strain gauge measurement cannot be doubled by ultrasonic method and the accuracy of the measurement was much lower than that achieved for moduli E and E'.

Eight standard cubical specimens of the ordinary concrete B20 have been uniaxially compressed in Instron testing machine of 1000 kN capacity. Longitudinal and lateral deformations of the specimen were measured by means of four electrical resistance strain gauges 1, 2, 3 and 4 arranged in the form of two element rosettes glued on the opposite sides of cubes as shown in Fig. 1. To minimise the influence of a friction, that exists at the contact surfaces between the specimen and the platens of the testing machine, on deformations measured in the central area of the specimen, two auxiliary cubes I and II were added. To assure the uniform distribution of stresses at the contact surfaces, two 3 mm thick pads p_1 and p_2, made of soft plywood, were placed between specimen tested and two auxiliary cubes. The arrangement of the specimen shown in Fig. 1a made it possible to measure the modulus of elasticity E' and relevant

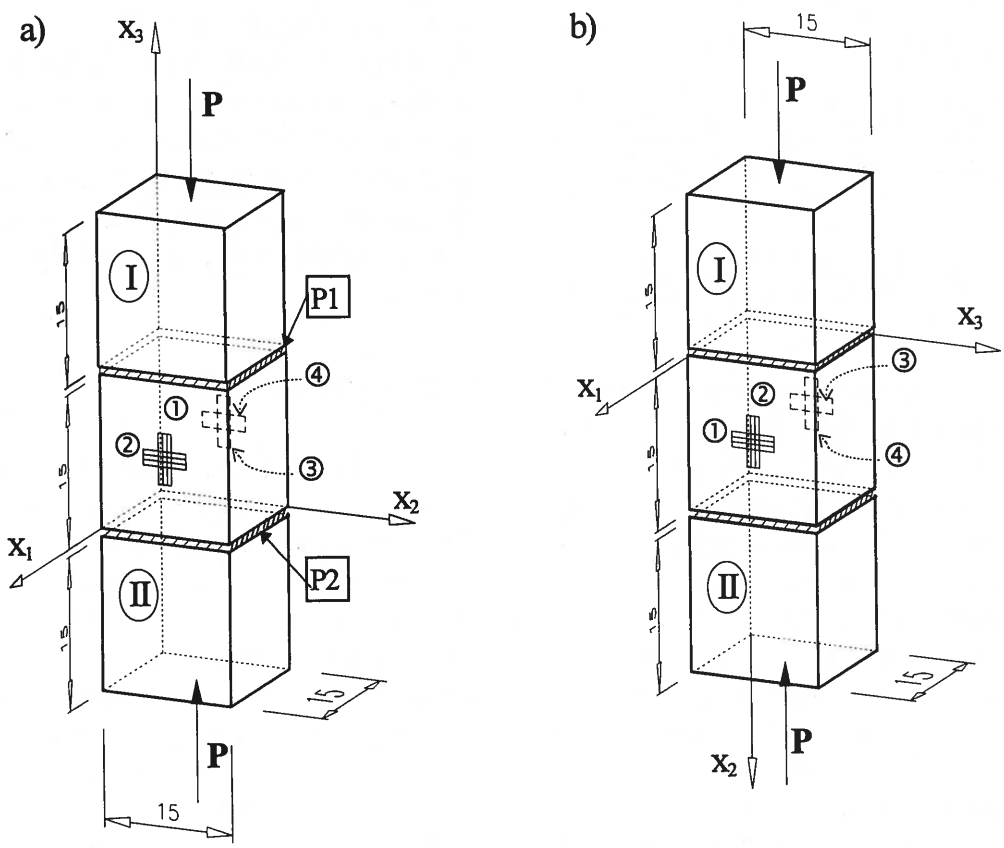

Fig. 1. Specimen arrangements: a) for longitudinal loading
b) for transverse loading

coefficient of lateral contraction ν'. To measure experimentally the modification of the modulus E, which in this case is identical in both lateral directions determined by the axes x_1 and x_2, the velocity of ultrasonic pulses that travelled in the direction x_2 were recorded simultaneously with the measurements of the other constant E'. Ultrasonic technique enables to measure dynamic modulus E_d by employing a formula

$$E_d = \rho V^2 \frac{(1+\nu)(1-2\nu)}{1-\nu} \qquad (12)$$

where V is a velocity of ultrasonic pulse, ρ is a density of the material and ν is a coefficient of lateral deformation. The dynamic modulus of elasticity E_d determined from the equation (12) is higher than the static modulus E which could be measured by using standard strain gauge experimental procedure. To obtain the static modulus of elasticity, various empirical formulas could be used like those, mainly linear equations, discussed in the monograph by Neville (1995). However, more realistic seems to be non-linear relation between these moduli and that is why the relevant graphs shown in the manual of Electronic Ltd (1991) were used to elaborate the experimental data obtained from ultrasonic measurements.

First phase of the experiments consisted of the ultrasonic pulse velocity measurements done in the three directions x_1, x_2 and x_3 for the specimens free of any load. The purpose of this tests was to check an initial isotropy of the material in its original state. Additionally, four specimens arranged as shown in Fig. 1b were subjected to initial compressive stress, that did not exceed 15% of the compressive strength of the material R_b , applied in the direction of x_2 axis in order to determine the relevant initial modulus of elasticity and coefficient of Poisson. The average values of the initial moduli of elasticity for the concrete tested, by means of both methods, in three directions, x_1, x_2 and x_3 were equal to E_1=19800 MPa, E_2=18700 MPa and E_3=19100 MPa, respectively. Insignificant diferences between these values show that the material tested is pratically isotropic in its initial state, having initial material constants E_0 = 19200 MPa and ν_0 = 0,20.

The objective of the second phase of the experiments was to determine the stress-strain curves as well as to measure the modification of the material constants for monotonically increasing load up to material failure. To this end four specimens were loaded as shown in Fig.1a and the constants E' and ν' were measured by employing the strain gauge measurements whereas the value of the modulus E was determined by means of ultrasonic method.

The experiments of the third phase were performed for those four specimens which have already been initially tested under small load applied in the direction of the x_2 axis. The loading programme for these four specimens consisted of four cycles, considering as a cycle 1 the compressive load in the direction of the x_2 axis that was applied in the first phase of the experiments. The cycle 2 consisted of the compressive load in the direction of the x_3 axis above the point of initial cracking of the concrete, and of unloading. Two specimens were loaded in this cycle to the value of $0{,}555R_b$ whereas the other two were loaded up to $0{,}825R_b$. The programme of measurement of the constants E',ν' and E was the same as that in the second phase of the experiment. This cycle of loading resulted in accumulation of oriented damage and in development of material anisotropy, smaller for the specimens subjected to lower load and more pronounced for those loaded to the level closer to the compressive strength R_b. To detect this damage induced anisotropy the specimens were loaded in the cycle 3 in the direction of the x_2 axis. The compressive stress of $0{,}15R_b$ applied in this cycle was too small to change the internal structure of the material but was sufficient to measure the constants E and ν by means of the strain gauges and also E' by employing the ultrasonic technique. Thus, the objective of this cycle of loading was to double the measurements done in the cycle 2, where the modulus E was measured by means of ultrasonic equipment but the constants E' and ν' were obtained from the strain gauge measurements. The cycle 4 was the continuation of the measurements done in the cycle 2 under the compressive load applied in the direction of the x_3 axis up to material failure.

The programme of loading in the cycles 2 and 4 made it possible to determine in each stage of load the values of the constants E' and ν' by employing the data obtained for the strain gauges and E from the measurements of the velocity of ultrasonic pulses. On the other hand, in the cycles 1 and 3 the constant E' was measured by means of ultrasonic method and the values of E and ν were obtained from the strains measured. It means that for three levels of loading determined by the compressive stress σ_3 equal to 0, $0.555R_b$ and $0.825R_b$, the measurements were doubled utilising both methods. The experimental results shown in Fig.2, 3 and 4, are the mean values obtained for all the cubes tested within the above programme of loading.

4. RESULTS AND DISCUSSION

Experimentally obtained stress-strain curves as well as the relevant data on material constants modification in the loading process were compared with the theoretical predictions represented by equations (1), (2) and (3). However, these constitutive equations contain four unknown constants A, B, C and D which should be identified experimentally. To this end the equation (6) was specified for two values of the compressive stress equal to $\sigma_3=mR_b$ and nR_b. Substituting the longitudinal strains ε_{3m} and ε_{3n}, which correspond to these load levels, the following two equations were obtained:

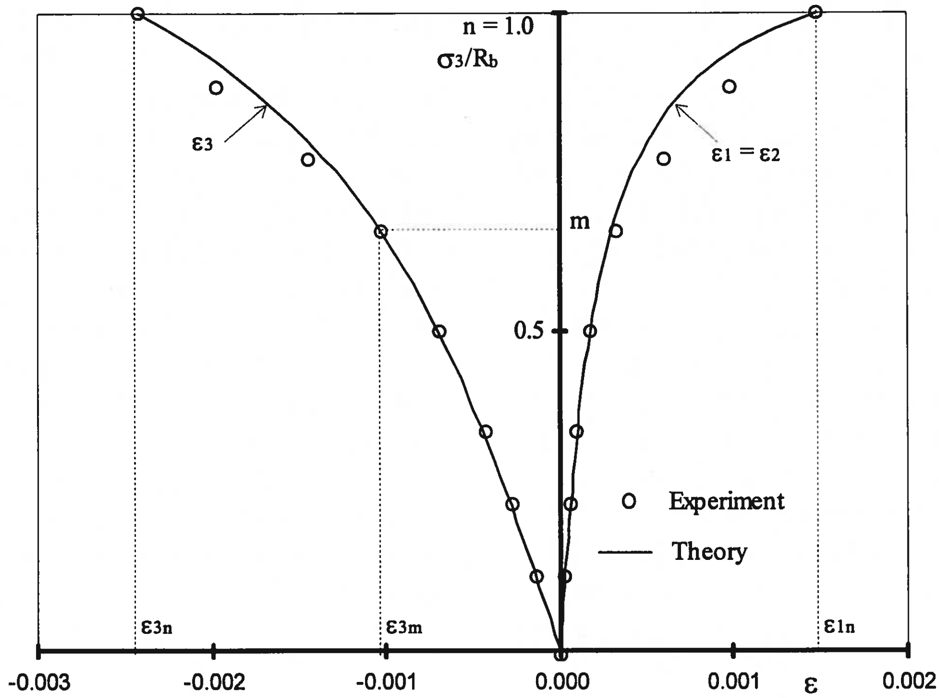

Fig.2. Longitudinal and transverse strain for B20 concrete versus longitudinal stress

$$2A-3B=\frac{3(1-C_{mn})}{(nR_b)^2-C_{mn}(mR_b)^2} \qquad (13)$$

$$C+2D=\left(\varepsilon_{3n}-\frac{nR_b}{E_0}\right)\frac{3-(2A-3B)(nR_b)^2}{2(2A-3B)(nR_b)^3} \qquad (14)$$

where, $$C_{mn}=\left(\frac{n}{m}\right)^3\frac{E_0\varepsilon_{3m}-mR_b}{E_0\varepsilon_{3n}-nR_b}$$

The next relation necessary to determine the unknown constant

$$A=\frac{3(E_0-E_n)}{2(nR_b)^2\left[2E_0E_n(C+2D)+E_0-E_n\right]} \qquad (15)$$

was obtained from equation (7), written for a compressive stress $\sigma_3=nR_b$ and corresponding modulus of elasticity E_n. The last

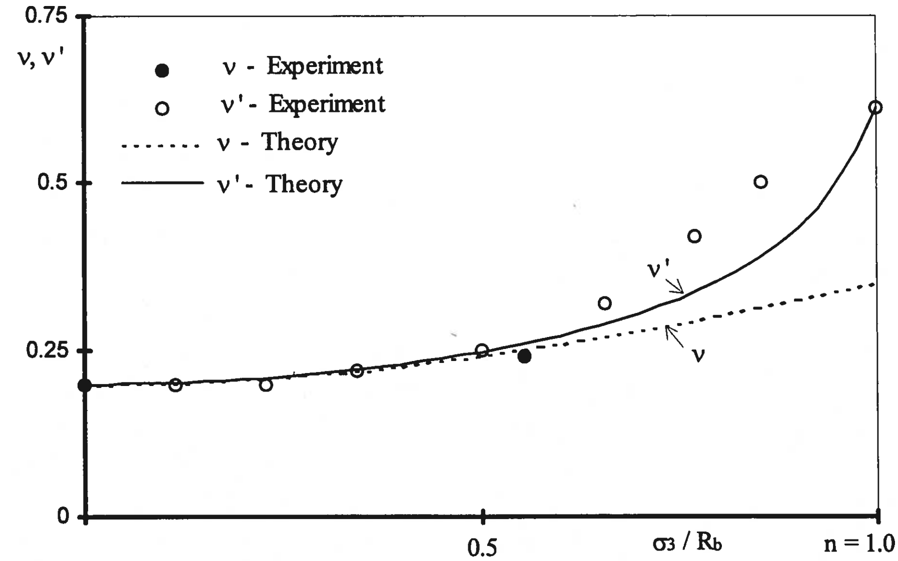

Fig.3. Modification of the coefficient of lateral deformation in loading process for B20 concrete

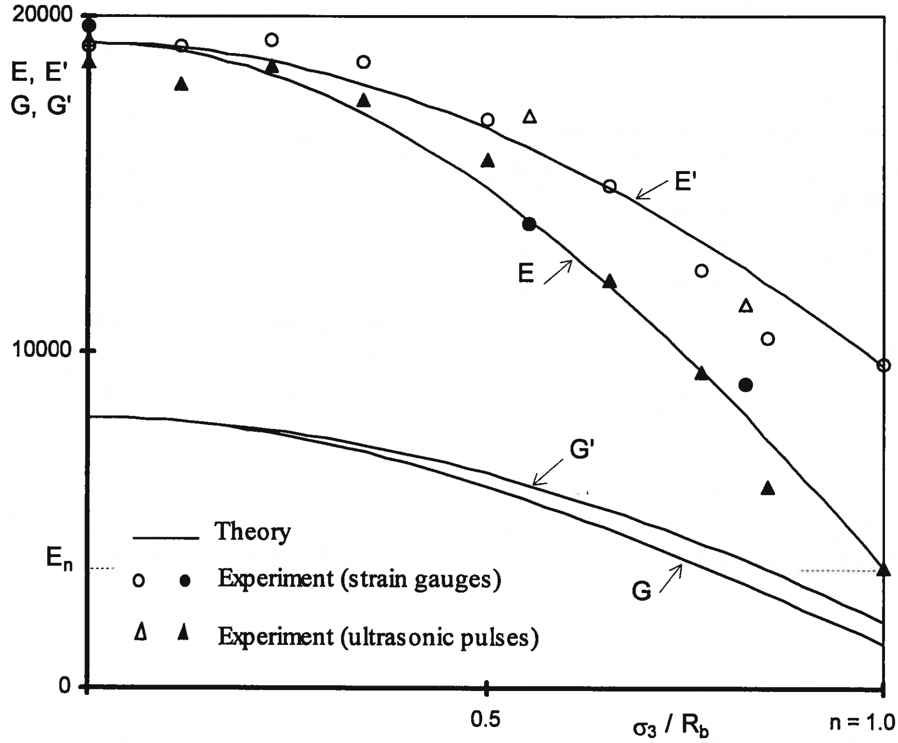

Fig.4. Modification of the moduli of elasticity in loading process of B20 concrete

constant can be determined from the formula

$$C=\left(\varepsilon_{1n}+\frac{v_0}{E_0}nR_b\right)\left[\frac{2A(nR_b)^3}{3-2A(nR_b)^2}+\right.$$

$$\left.+\frac{2A(nR_b)^3-3B(nR_b)^3}{3-2A(nR_b)^2+3B(nR_b)^2}\right]^{-1} \qquad (16)$$

which is the equation (5) expressed for σ_3=nRb and respective value of the transverse strain ε_{1n}.

The values of the constants A = 218x10^{-5} MPa^{-2}, B = 60.8x10^{-5} MPa^{-2}, C = -1.20x10^{-5} MPa^{-1} and D = 2.17x10^{-5} MPa^{-1} were calculated from equations (13), (14), (15) and (16) by employing the following data: E_0 = 19200 MPa, v_0 = 0,20, R_b = -23.2 MPa, n = 1, m = 0.656, ε_{3n} = -0.002424 , ε_{3m} = -0.001025 , ε_{1n} = 0.001483 and E_n = 3600 MPa.

The comparison of the theoretical stress-strain curves obtained from equations (5) and (6) as well as the curves showing the modification of the material constants E, ν, E', ν' and G', determined by equations (7), (8), (9), (10) and (11), with the respective experimental results is shown in Figs. 2, 3 and 4. The additional curve of the relation between the constant $G=\frac{E}{2(1+\nu)}$ and load level $\frac{\sigma_3}{R_b}$ is shown in Fig.4 in order to show the difference between two shear moduli G and G'.

5. CONCLUSIONS

The experiments performed made it possible to study quantitatively the damage induced anisotropy of the concrete. Initially isotropic material when uniaxially compressed becomes transverse isotropic one due to oriented damage that grows in the loading process. Moreover, it was proved experimentally, that values of the moduli of elasticity measured by employing the ultrasonic technique are practically the same as those determined by means of the electric-resistance strain gauges.

Comparison of the experimental results with the theoretical predictions enabled the verification of the mathematical model proposed. Theoretical stress-strain curves as well as the relations showing the modification of the material constants in the loading process coincide with experimental data obtained. It means that experiments performed corroborate the validity of the constitutive equations developed for the concrete on the basis of continuous damage mechanics.

6. ACKNOWLEDGEMENTS

This work was done within the JNICT Programme, F.P. da U. I. & D Nr. 202 and P.U.T. Programme D.S./1997.

7. REFERENCES

Eimer, C. (1971), "Rheological Strength of Concrete in the Light of Damage Hypothesis", Archives of Civil Engineering, Nr. 1, vol. 17, 15-31 (in Polish).

Spencer, A.J.M. (1971), "Theory of Invariants", in: Continuum Physics, ed. C. Eringen, Academic Press, New York, vol. 1, 239-353.

Geniev, G.A., Kissiuk, V.N. and Tiupin, G.A. (1974), "Theory of Plasticity of Concrete and Reinforced Concrete", Stroiizdat, Moscow (in Russian).

Dragon, A. (1976), "On Phenomenological Description of Rock-Like Materials with Account for Kinetics of Brittle Fracture", Archives of Mechanics, Nr.1, vol.28, 13-30.

Murakami, S. and Ohno, N. (1981), "A Continuum Theory of Creep and Creep Damage", in: Creep in Structures, eds. A.R.S. Ponter, D.R. Hayhurst, Springer Verlag, Berlin, 422-444.

Chen, W.F. (1982),"Plasticity of Reinforced Concrete", McGrow-Hill, NewYork.

Betten, J. (1983), "Damage Tensors in Continuum Mechanics", Journal de Méchanique Théorique et Appliquée, Nr.1, vol. 2, 13-32.

Boehler, J.P., ed. (1987), "Applications of Tensor Functions in Solid Mechanics", Springer Verlag, Wien.

Karpenko, N.J. (1987), "On Formulation of General Orthotropic Model of Concrete Deformability", Stroitielnaia Mekhanika Rascot Sooruzhenij, Nr. 2, 31-36 (in Russian).

Baikov, V.N. (1990), "Particular Features of Concrete Fracture Due to Its Orthotropic Deformation", Beton Zelezobeton, Nr.9, 19-21 (in Russian).

Electronics Ltd, C.N.S. (1991), "Pundit Manual for Use with the Portable Ultrasonic Non-Destrutive Digital Indicating Tester", London.

Litewka, A. (1991), "Creep Damage and Creep Rupture of Metals", Wyd. Politechniki Poznanskiej, Poznan.

Mitrofanov V. P. and Dovzenko O.A. (1991), "Development of Deformation Induced Anisotropy of Concrete under Uniaxial Compression", Beton Zelezobeton, Nr. 10, 9-11 (in Russian).

Neville, A. M. (1995), "Properties of Concrete", Longman, Harlow.

Litewka, A. Bogucka, J. and Debinski, J. (1996), "Deformation Induced Anisotropy of Concrete", Archives of Civil Engineering, Nr. 4, vol. 42, 425-445.

Litewka, A. , Debinski, J. (1997), "Damage Induced Anisotropy of Concrete Subjected to Multi-Axial State of Stress", Mathematical Modelling and Scientific Computing, vol. 8 (in press).