# CONTINUOUS DYNAMIC MONITORING OF LARGE CIVIL INFRASTRUCTURES

# MONITORIZAÇÃO DINÂMICA EM CONTÍNUO DE GRANDES INFRASTRUTURAS DE ENGENHARIA CIVIL

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### ABSTRACT

This paper briefly describes a set of long-term dynamic monitoring programs developed by the Laboratory of Vibrations and Structural Monitoring (ViBest, www.fe.up.pt/vibest) of CONSTRUCT/FEUP in large Civil structures with different typologies (e.g. roadway, railway and pedestrian bridges, stadia suspension roofs, wind turbines, concrete dams or high voltage transmission lines), showing the interest and potential of the developed technology, as well as of the huge high quality database created, which can be used for joint collaborative research at European level.

### RESUMO

Este artigo descreve de forma sucinta um conjunto de programas de monitorização dinâmica permanente desenvolvidos pelo Laboratório de Vibrações e Monitorização de Estruturas (ViBest, www.fe.up.pt/vibest) do CONSTRUCT/FEUP em grandes estruturas de Engenharia Civil com diferentes tipologias (e.g. pontes rodoviárias, ferroviárias e pedonais, coberturas suspensas de estádios, turbinas eólicas, barragens de betão ou linhas de muito alta tensão), evidenciando o interesse e potencial da tecnologia desenvolvida, bem como a enorme base de dados de alta qualidade constituída, a qual pode ser utilizada no âmbito de investigação colaborativa a nível Europeu.

<u>Keywords</u>: Monitorização Dinâmica, Pontes, Estádios, Torres Eólicas, Barragens, Linhas de Alta Tensão

## **1. INTRODUCTION**

The Laboratory of Vibrations and Monitoring (ViBest, <u>www.fe.up.pt/vibest</u>) of CONSTRUCT/ FEUP has been implementing, since 2007, a significant set of longterm dynamic monitoring systems in large Civil structures with different typologies (e.g. roadway, railway and pedestrian bridges, stadia suspension roofs, wind turbines, concrete dams or high voltage transmission lines).

This paper briefly describes some of these applications, clearly illustrating the interest and potential of the developed technology, as well as of the huge high quality database created, which can be used for joint collaborative research at European level.

The representative set of monitoring applications presented shows the efficiency of the developed tools and the usefulness of the testing and monitoring programs implemented, enabling the achievement of different objectives, such as: (i) the development of finite element model correlations and updating; (ii) the vibration serviceability safety checking, particularly in case of

lively bridges involving the inclusion of vibration control devices; (iii) the implementation of automated versions of the most powerful methods of Operational Modal Analysis, and their application for tracking the time evolution of modal parameters in long-term dynamic monitoring applications; (iv) the application of statistical methods to remove the influence of environmental and operational factors (e.g. temperature, intensity of traffic, wind) on the modal variability, supporting the development of reliable techniques for vibration based damage detection; (v) the experimental assessment of fatigue, based on the measurement of effects of real traffic loads; (vi) the experimental assessment of aerodynamic problems in bridges based on in-situ measurements; (vii) the tracking of modal parameters in wind turbines, enabling damage detection and fatigue assessment and (viii) the characterization of the influence of the water level in the reservoir on the dynamic properties of concrete arch dams.

### 2. MONITORING OF HUMAN INDU-CED VIBRATIONS FOR VIBRA-TION SERVICEABILITY SAFETY CHECKING

The continuous dynamic monitoring of human induced vibrations can be useful to support the verification of safety with regard to vibration serviceability limit states in lively structures and of the efficiency of vibration control devices.

This is the case, for instance, of Pedro e Inês footbridge (Fig. 1(a)), where ViBest/

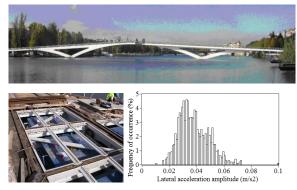


Fig. 1 - (a) Lateral view of Pedro e Inês footbridge;
(b) TMD units for control of lateral vibrations; (c)
Histogram of maximum daily lateral accelerations (June 2007 – May 2010).

FEUP implemented a long-term continuous dynamic monitoring to check the real efficiency of a set of tuned mass dampers (TMDs, Fig. 1(b)) installed at different points of the deck to control excessive lateral and vertical accelerations.

This bridge is a slender structure 275m in length and 4m wide, except in the central square with dimensions of 8mx8m. The metallic arch spans 110m and rises 9m and has a rectangular box cross-section with 1.35m x 1.80m. The deck has a L-shaped box cross-section, the top flange being formed by a composite steel-concrete slab 0.11m thick. In the central part of the bridge, each L shaped box cross-section and corresponding arch "meet" to form a rectangular box cross-section 8m x 0.90m. In the lateral spans, arch and deck generate a rectangular box cross-section 4m x 0.90m. The significant slenderness of the bridge and the geometric characteristics lead to a complex structural behaviour. A key factor for the bridge global stiffness is the structural behaviour of its foundations, which are formed by vertical piles deep about 30m.

Dynamic studies developed at the design stage could anticipate that this bridge would be prone to high levels of vibration induced by pedestrians, owing to the existence of a fundamental lateral bending mode with a frequency close to one half of the mean pacing rate, easily inducing a lateral synchronization phenomenon (lock-in), as well as several natural frequencies associated to vertical bending modes susceptible to be excited by pedestrians walking, running or rhvthmic jumping. Therefore. under ViBest/ FEUP, under contract with the designer (AFAssociados), and subsequently with the construction company (Soares da Costa/ Socometal), developed during the design phase a preliminary design of a set of TMDs to control the critical modes of vibration. After the end of construction, it was made a final design, which required the performance of a series of dynamic tests (ambient and free vibration tests and tests with pedestrians), which enabled an accurate updating and experimental validation of a sophisticated finite element model, that was used for an accurate evaluation of the modal masses in correspondence with the modes of vibration to be controlled. All this extensive work is described in detail in [1][2]. After installation of TMDs, forced vibration tests were also performed to accurately measure the levels of damping achieved. In the meanwhile, the owner also required a continuous dynamic monitoring of the footbridge during a period of 5 years to confirm the good performance of the vibration control devices used.

The monitoring system, described in detail in [2], is composed by 6 piezoelectric accelerometers, a signal conditioning system and an industrial PC, with an ADSL connection to the Internet. Every 20 minutes, all the collected time series are transmitted to FEUP, becoming available to the constructer, designer and owner through a website supported by a LabVIEW application, and allowing to issue alert messages in case any lateral or vertical vibration comfort level is exceeded.

Fig. 1(c) shows, for instance, a histogram associated to maximum daily lateral accelerations at midspan during a period of 3 years, which clearly shows that, after implementation of the lateral TMD, no situation of excessive lateral vibrations was recorded in that period.

#### 3. MONITORING OF DYNAMIC EFFECTS OF TRAFFIC LOADS IN ROADWAY AND RAILWAY BRIDGES

The dynamic monitoring of bridges also enables an accurate evaluation of dynamic effects induced by traffic loads on roadway or railway bridges.

This kind of study was for instance developed by ViBest/ FEUP on Salgueiro Maia bridge (Fig. 2a)), a single-plane cablestayed bridge over Tagus river, upstream the Vasco da Gama bridge. The main objective of this study, described with more detail in [3][4][5], was the experimental evaluation of dynamic effects induced by the passage of heavy trucks using a monitoring system based on strain gages embedded in the concrete and load cells in the staycables.

This system was used during the static

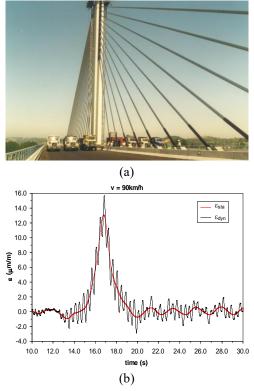


Fig. 2 - (a) Salgueiro Maia bridge during the static load tests; (b) Record from embedded strain gage (vehicle speed of 90 km/h).

load tests involving 20 heavy trucks with total mass of 38-40t each in different loading configurations. Afterwards, dynamic tests were performed in which some of those trucks crossed the bridge at different velocities (15, 30, 45, 60, 75 and 90km/h), and along different lanes, alone or in groups. As an example, Figure 2b) shows a dynamic response recorded during the passage of a heavy truck crossing the bridge at 90km/h along a lateral lane, as well as the quasi-static response obtained by digital filtering. The dynamic amplification factors obtained (DAFs) reached, in this case, about 1.20. It's worth noting however that the induced dynamic effects are significantly dependent upon the pavement roughness conditions, which were characterized in this case by a spatial laser scanning.

Another interesting example regards the experimental evaluation of the dynamic effects induced by traffic loads on the Trezói railway bridge (Fig. 3). This bridge is a single track metallic riveted structure with three spans of 39m, 48m and 39m, supported by two intermediate trapezoidal metallic piers and two masonry abutments at the extremities.



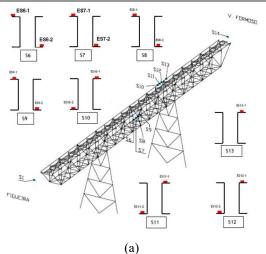
Fig. 3 - Trezói bridge: (top) lateral view; (bottom) view inside the deck.

A temporary monitoring campaign (Fig. 4) [6] was also developed at this bridge by ViBest/ FEUP to characterize the global and local structural behaviour, enabling the calibration and validation of numerical models used in numerical simulations.

#### 4. MONITORING FOR FATIGUE ASSESSMENT OF RAILWAY BRIDGES AND WIND TURBINE TOWERS

The temporary monitoring campaign of Trezói railway bridge was also used for the analysis of stress distributions in critical elements and connections, the construction of histograms of stress cycles suitable for fatigue assessment, and collecting reliable information regarding the characteristics of real traffic crossing the bridge in terms of velocity, axle loads, number of axles and axles distances [6][7].

In a first instance, measurements were performed during two days of higher level of traffic. During that period, the bridge was crossed by 8 freight trains and 16 passenger trains. Fig. 4b) shows the very good agreement between measured and calcula-



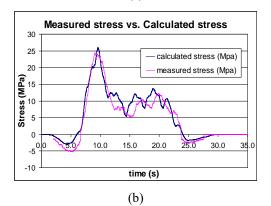


Fig. 4 - (a) Instrumentation with strain gages; (b) Measured vs calculated stresses.

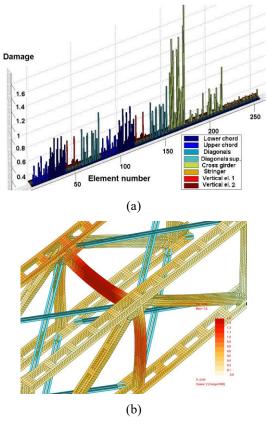


Fig. 5 - (a) Evaluation of fatigue damage for real traffic; (b) Shell finite element modelling.

ted stresses at an inferior chord of the truss at midspan, which is the element subjected to higher traction stresses. These results provided a higher confidence on the developed numerical simulations, as well as on the trains characteristics obtained. Fatigue damage was then evaluated for the 14 instrumented sections and for each train.

This investigation was subsequently extended to all structural elements of the bridge, which led to the conclusion that the most susceptible elements to fatigue damage are the transversal stringers (Fig. 5), which are affected by local vibrations. This aspect motivated a new research component specifically focused on these local effects, which involved also the development of more sophisticated numerical models (Fig. 5b)) and the continuous dynamic monitoring of two transversal stringers [6][7].

A different approach has been also developed for the fatigue assessment of wind turbine towers essentially based on the use of accelerometers [8]. This methodology, illustrated and validated with the help of a numerical example developed with the HAWC2 code, is based on the decomposition of the acceleration data records in modal acceleration responses. After this step, the fatigue stress due to each vibration mode or stationary responses to harmonic excitations is computed.

#### 5. DYNAMIC MONITORING FOR MODAL VARIABILITY ANALYSIS OF BRIDGES, STADIA SUSPEN-SION ROOFS, WIND TURBINES AND CONCRETE DAMS

Continuous dynamic monitoring can also be used to evaluate the modal variability induced by environmental and operational factors, such as temperature, traffic intensity, wind speed or water level in reservoirs.

For instance, a 12-channel continuous monitoring system installed at Infante D. Henrique bridge (Figure 6) [9], in Porto, Portugal, in 2007, has been used to track since then the time evolution of the corresponding modal estimates, by automated processing of the acceleration time series collected every half an hour using very robust home-made software (DYNAMO) for output-only modal identification based on the most advanced methods (SSI and p-LSCFD) [10][11].

Figure 7 shows the time evolution of the estimates of the first 12 natural frequencies along one year, by application of the p-LSCF method. Though these estimates look rather stable at a first glance, an appropriate zoom of this plot for each natural frequency clearly shows the existence of daily and seasonal variations (Figure 8), which are mainly motivated by oscillations of temperature and intensity of traffic.

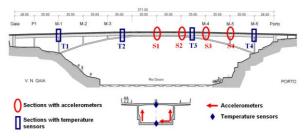
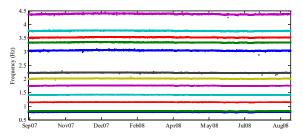


Fig. 6 - Position of accelerometers and temperature sensors at Infante D. Henrique bridge.



**Fig. 7** - Time evolution of the estimates of the first 12 natural frequencies along one year, by application of the p-LSCF method.

Figure 9 (top) shows, for instance, the good correlation between the estimates of the first natural frequency and measured temperature at the top of section T3. On the other hand, Fig. 9 (bottom) illustrates the influence of the increase of traffic intensity at the beginning of each morning on the estimates of the second natural frequency.

In structures with clear linear behaviour under ambient excitation, as is the case of Infante D. Henrique bridge, the increase of temperature usually induces a decrease of natural frequencies according to some linear relation. However, in some structures some form of non-linear behaviour can cause a non-linear relation. This fact can stem either from some type of material nonlinearity (e.g. sudden variation of the stiffness of the asphalt layer for negative temperatures) or from different forms of geometrical non-linearity (e.g. induced by temperature joints or long suspension cables).

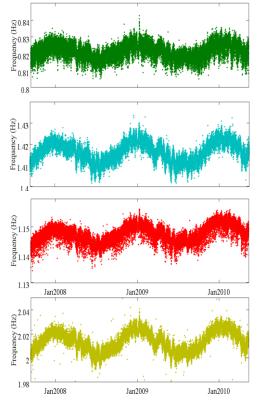
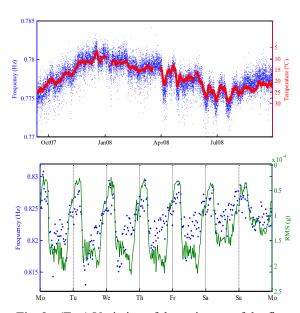


Fig. 8 - Time evolution of the estimates of the first 4 natural frequencies along three years, by application of the p-LSCF method.



**Fig. 9** - (Top) Variation of the estimates of the first natural frequency (blue dots) vs measured temperature at the top of section T3 (red line); (Bottom) Time evolution of the second natural frequency vs RMS values of vertical acceleration time series collected at section S3 during one week (03/03/2008 - 09/03/2008).

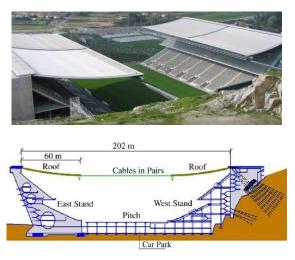
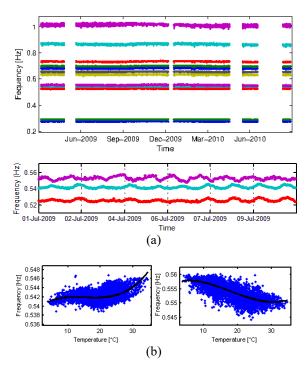


Fig. 10 - Braga stadium suspension roof: View from the west side; Cross-section.

As an example, we can mention the case of the Braga Stadium suspension roof (Figure 10), where a 6-channel continuous dynamic monitoring system has been installed in 2009, enabling the tracking of a large number of modal frequencies in the range 0-1 Hz (Figure 11) [12]. However, careful inspection of the zoom shown in Figure 11 shows the occurrence of some non-linear correlation between frequencies and temperature induced by the non-linear geome-



**Fig. 11** - (a) Temporal evolution of identified natural frequencies: (top) from 25/03/2009 to 27/09/2012 in the range 0-1.1 Hz; (bottom) from 1/07/2009 to 10/07/2009 for modes 3, 4 and 5; (b) Correlation between modal frequencies and measured temperatures in the period 1/07/2009 to 31/12/2009.

trical characteristics of the suspension roof, and in some cases the increase of temperature leads even to an increase of modal frequency [13][14].

Wind can also play an important role in terms of inducing significant variations in the modal parameters. This has been noticed for instance processing data from one year of continuous monitoring at Torrão wind turbine [8] and observing the Campbel diagram achieved relating frequency estimates with the rotor speed (Figure 12). It is however particularly interesting to show the significant increase of modal damping of the 1st FA mode induced by the increase of wind speed along the different operating regimes of the wind turbine (Figure 13).

At last, it's still worth mentioning the influence of the water level in the reservoir on the natural frequencies of Baixo Sabor dam recently detected by processing the data from a 20-channel continuous dynamic monitoring system installed after construction to follow the first filling of the reservoir (Figure 14) [15].

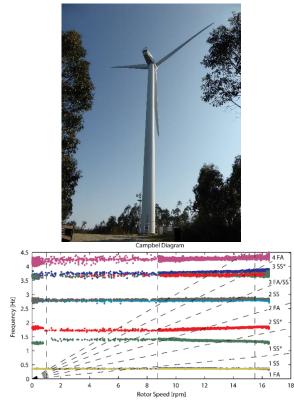


Fig. 12 - (Top) Torrão wind turbine; (Bottom) Campbel diagram after one year of continuous monitoring and removal of the estimates corresponding to excitation harmonics.

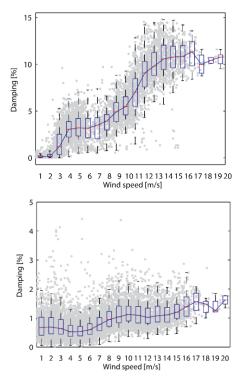


Fig. 13 - Evolution of damping with the wind speed: (Top) 1st FA tower mode; (Bottom) 1st SS tower mode.

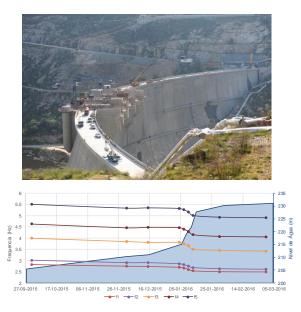


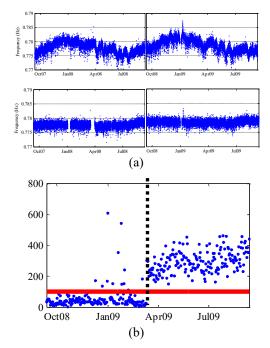
Fig. 14 - Variation of identified natural frequencies of Baixo Sabor dam with the water level in the reservoir.

#### 6. MONITORING FOR VIBRATION BASED EARLY DETECTION OF STRUCTURAL CHANGES

Although the variations of modal frequency estimates induced by environmental and operational factors are in general relatively small, they can clearly disturb any attempt of detecting structural damages on the basis of the analysis of natural frequency shifts.

Therefore, it is of utmost importance to remove or mitigate the effect of such factors on the modal variability by applying appropriate statistical methods (e.g. multiple linear regression or PCA methods) and building suitable control charts that may flag the occurrence of slight damage [16][17].

Figure 15(a) shows, for instance, the temporal evolution of the estimates of the first natural frequency of Infante D. Henrique bridge before and after the removal of the environmental/ operational effects. Inspection of this figure shows the high efficiency of such correction, most of the estimates being then located in a very narrow frequency range with an amplitude of about 0.005 Hz. This means that relatively small damage can be detected in the future provided that the variations of modal frequencies induced are higher than that order of magnitude. This conclusion has been better confirmed analysing different damage scenarios idealized numerically and building appropriate control charts, as illustrated in Figure 15(b) [16][8], both in bridges and in wind turbines.



**Fig. 15** - (a) Temporal evolution of the first natural frequency estimates before and after the removal of environmental/ operational effects; (b) Control chart for damage detection.

#### 7. DYNAMIC MONITORING OF WIND EFFECTS ON LARGE BRIDGES AND SUSPENSION ROOFS

Continuous dynamic monitoring can also play an interesting role in Wind Engineering studies developed on prototypes, specially in the case bridges located in zones with high susceptibility to the occurrence of extreme events, such as cyclones or typhoons, demanding an adequate characterization of loads and an accurate assessment of the corresponding response.

Such type of research has been carried out by ViBest/ FEUP on the Grande Ravine viaduct (Fig.16), which is a slender girder bridge crossing a volcanic breach of 320m width and 170m depth, at the Reunion Island. This viaduct is located in an area frequently affected by tropical cyclones and has a particular structural configuration that, although stiff when compared to cablestayed and suspended bridges, is slender in comparison to ordinary girder bridges.

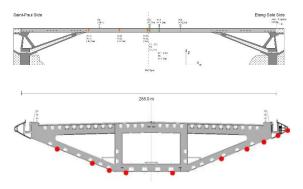


Fig. 16 - Monitoring of Grande Ravine viaduct: Location of (top) anemometers and accelerometers; (bottom) pressure cells.

These aspects led the Designer SETEC tpi [18] to idealise a continuous monitoring system with aerodynamic character, grouping a set of anemometers, pressure cells, temperature sensors and accelerometers (Fig.16). Collaboration between the Designer, ViBest/FEUP and CSTB was established for the processing and analysis of recorded data, in order to validate the wind studies carried out during the design stage. Several other experimental tests were also conducted. In particular, ambient vibration tests were performed to characterise the bridge structure after construction, and detailed wind tunnel tests were developed, using simultaneous measurement of aerodynamic forces with balance and through pressures on the surface of cross section of the deck model, in order to characterise the aerodynamic forces acting on the viaduct model [19].

#### 8. DYNAMIC MONITORING OF HIGH VOLTAGE TRANSMISSION LINES

The wind excitation on high voltage transmission lines is responsible by significant problems induced by vibrations, causing conductors ruptures and damages in isolators and dampers, which can lead to disturbances in the electric energy supply.

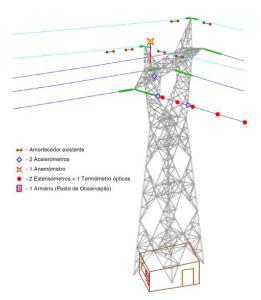


Fig. 17 – Location of instrumented sections

The necessity to characterise the life time of an electric conductor based on its structural health condition, as well as to evaluate the efficiency of currently available vibration mitigation measures, is the origin project of the research DYNCATLINE [20], developed in cooperation with Rede Elétrica Nacional (REN). This research involved the continuous dynamic monitoring of a large span of a high voltage transmission line during a period of two years. During that period, it was possible to measure the wind excitation and the corresponding structural response in several points along an electric conductor and at the top of a tower.

It was possible to characterise the dominant characteristics of wind in terms of speed, direction, incidence, as well as the levels of ambient response, vertical and transversal with regard to the line, identifying situations susceptible to induce higher amplitudes of vibration. The comparison of the structural response before and after the installation of "Stockbridge" dampers also allowed to evaluate their level of efficiency [21].

#### 9. CONCLUSIONS

This paper briefly refers a set of longterm dynamic monitoring systems implemented by the Laboratory of Vibrations and Structural Monitoring (ViBest) of FEUP in large Civil structures with different typologies (e.g. roadway, railway and pedestrian bridges, stadia suspension roofs, wind turbines, concrete dams and high voltage transmission lines). These applications clearly illustrate the interest and potential of the developed technology in different perspectives, as well as the huge high quality database created, which can now be used for joint collaborative research at European level.

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