VARIABLE AMPLITUDE FATIGUE CRACK GROWTH MODELLING

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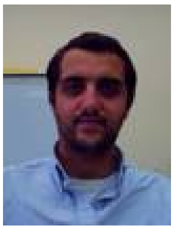

ABSTRACT

Fatigue crack growth in structural components is often subjected to variable amplitude loading. This paper describes the more relevant crack growth transients generally observed under simple variable amplitude loading sequences and several crack propagation models that consider the load interaction effect in fatigue crack growth. Many models have been proposed to predict fatigue crack propagation that consider the loading history. The basic models are analysed and discussed. The Wheeler model is one of the simplest and most widely employed models to quantify the fatigue crack growth retardation after a single overload. This model is able to describe the basic phenomena of retardation due to overloads. However, due to its simple formulation is unable of accounting the other observed post-overload crack growth transients, the effect of underloads, and more difficulties arise when both overloads and underloads are involved. Therefore, more recent models that propose a number of modifications to the Wheeler model, to improve its accuracy in predicting fatigue crack growth under variable amplitude loading, are also analysed.

1 - INTRODUCTION

For many fatigue critical parts of structures, fatigue crack propagation under service conditions generally involves random or variable amplitude, rather than constant amplitude loading conditions.

Crack growth in structures is mainly influenced by the load amplitude and stress ratio. Under variable amplitude loading the load history or load sequence effect is also a major factor in determining fatigue life. Due to the random nature of variable loading, significant accelerations and/or retardations in crack growth rate can occur. Thus, an accurate prediction of fatigue life requires an adequate evaluation of these load interaction effects.

Overloads are known to retard crack growth, while underloads generally accelerate crack growth relatively to the baseline crack growth rate. These interactions, which are highly dependent upon the loading sequence, make the prediction of fatigue life under variable amplitude loading more complex than under constant amplitude loading.

Many models have been developed to predict the fatigue life of components under variable amplitude loading, which try to correctly evaluate the load interaction effects in crack growth propagation (Wheeler 1972; Willemborg 1971; Elber 1972; Newman 1981).

Generally, these retardation models may be classified into two main categories: crack tip plasticity models and crack closure models. The crack tip plasticity models are based on the assumption that crack growth retardation occurs due to the large plastic zone developed during overloading. The residual compressive stresses formed in this zone will reduce the magnitude of the tensile stresses during the next fatigue cycle and tend to delay crack growth. A number of retardation models belong to this category. The second main category of retardation models, the crack closure models, are based on Elber's experimental observation (Elber 1972) that, as a result of the tensile plastic deformation left in the wake of a fatigue crack, a partial closure of the crack faces occurs during part of a fatigue load cycle. Since crack propagation can only occur during the time for which the crack is fully open, the formation of crack closure reduces the range of the applied stress that is effective for crack growth.

More recently, models that include combinations of the Wheeler model with the strip yield model initially proposed by Newman (Huang *et al* 2005a), models based on the strain energy density factor (Huang *et al* 2005b), two parameter driving force models (Mikheevskiy and Glinka 2009) and others, have been proposed. However, due to the number and complexity of the mechanisms involved in this problem, no universal model exists yet.

The purpose of this paper is to describe and analyse the more relevant crack growth transients generally observed under simple variable amplitude loading sequences and several crack propagation models that consider the load interaction and stress ratio effects in fatigue crack growth under variable amplitude loading, using an equivalent stress intensity factor range.

2- LOAD INTERACTIONS EFFECTS UNDER SIMPLE VARIABLE AMPLITUDE LOAD SEQUENCES

Skorupa (1998) introduces the load interaction terminology to characterize the crack growth under variable amplitude loading, meaning that crack growth increment for a given cycle of a variable amplitude loading may be different than expected if constant amplitude is applied. A measure of average load interaction effects in crack growth can be estimated using the value of $\Sigma n/N$ which represents the ratio between the actual crack propagation life and that predicted from Miner's law. Instantaneous load introduction effects can be simulated by the ratio between the constant amplitude fatigue crack growth rate, $(da/dN)_{CA}$, and the measured under variable amplitude loading tests, $(da/dN)_{VA}$. Thus a $\Sigma n/N$ value or $(da/dN)_{CA}/(da/dN)_{VA}$ ratio larger than unity implies beneficial load interaction effects, occurring retardation of crack growth, whereas values less than unity indicate crack growth acceleration.

As already mentioned, an accurate prediction of fatigue life requires an adequate evaluation of the basic load interaction effects. To attain this objective several type of simple variable amplitude load sequences must be analysed and modelled.

Fig. 1 depicts the typical transient crack growth behaviour following a single tensile overload.

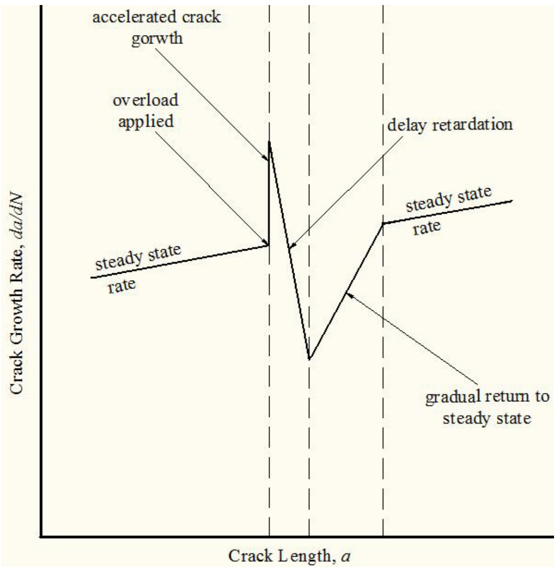

Fig. 1 – Typical crack growth transients following a peak overload (Yuen and Taheri 2006).

For a number of materials, namely for steels and aluminium alloys, it has been observed that an initial acceleration of the fatigue crack growth rate occurs immediately after an overload. The subsequent crack growth rate decreases until its minimum value is reached at some point ahead of the overload application, followed by a gradual approach to the constant amplitude crack growth level (Shin and Hsu 1993; Borrego *et al* 2001 and 2003). The observed behaviour is usually referred to as delayed retardation of crack growth.

The typical effect of periodically applied overloads for several numbers of baseline cycles between overloads can be seen in Fig. 2, for 2024-T3 aluminium alloy (Ohrloff *et al* 1988).

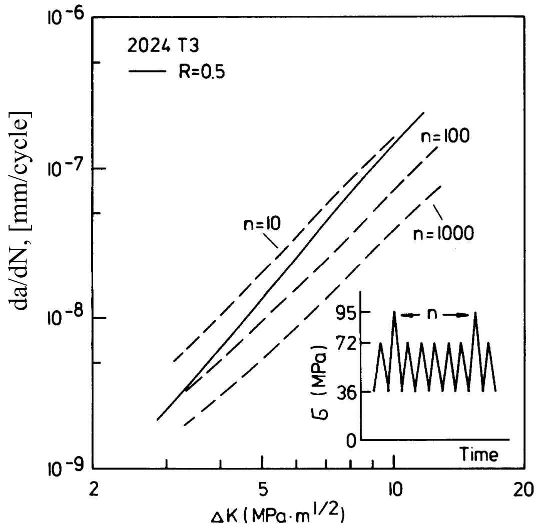

Fig. 2 - Crack growth rate behaviour under periodic overloads (Ohrloff *et al* 1988).

This figure shows that for remotely spaced overloads (n≥100 cycles) crack retardation and a corresponding decrease of *da/dN* relatively to constant amplitude loading are generally observed. In contrast, the results for more frequently applied overloads (n=10 cycles) present crack acceleration relatively to constant amplitude loading.

A reapplication of an overload after a period of baseline cycling reactivates mechanisms which lead to fatigue crack growth retardation. The most effective retardation is obtained when the period between overloads is sufficiently long to cause the crack growth rate to reach a minimum value. Additionally, at low ΔK baseline levels, when the overload reapplication occurs still during the acceleration stage associated with the prior overload, the overall effect can even be acceleration in fatigue crack growth rates (Ohrloff *et al* 1988; Borrego *et al* 2005).

Besides overloads and underloads the simple variable amplitude load sequences most commonly used to study the load interaction phenomenon are high-low (Hi-Lo), low-high (Lo-Hi) and change in stress ratio block load sequences as schematically represented in Fig. 3.

Fig. 4 illustrates the typical transient crack growth behaviour obtained following Hi-Lo and Lo-Hi block sequences under constant ΔK conditions.

The effect of the Hi-Lo block is similar to that observed for the peak overload. However, for this load sequence, the retardation is always immediate and not preceded by the acceleration phase (Ng' Ang'a and James 1996; Borrego *et al* 2008). The Lo-Hi sequence produces an acceleration of crack growth rate, above the steady state level expected for the high block, followed by a gradual reduction to the corresponding steady-state. ΔK_2 level. This behaviour is identical to that generally observed following an underload.

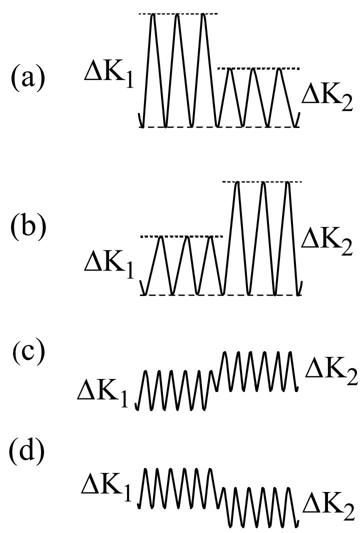

Fig. 3 - Schematic representation of two-level block loading: a) high-low; b) low- high; c) and d) Step in stress ratio.

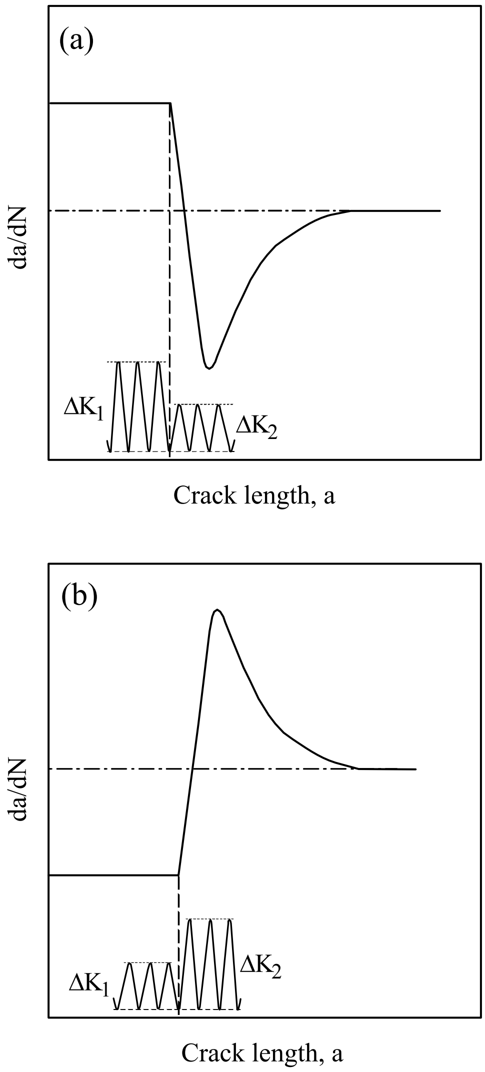

Fig. 4 - Schematic representation of crack growth behaviour following two-level block loading: a) Hi-Lo block; b) Lo-Hi block.

Relatively to the loading sequences (c) and (d) depicted in Fig. 3, the influence of the mean stress on the transient crack growth behaviour following a load step in Hi-Lo and Lo-Hi block sequences shows that the crack growth increment affected by the step in load is increased, although only slightly, when the stress ratio increases. However, a significant reduction of the delay cycles with increasing R is generally observed (Borrego *et al* 2008). Therefore, similar to the generally observed behaviour in tensile overloads, in steels (Shin and Hsu 1993) as well as in aluminium alloys (Borrego *et al* 2003), the retardation effect is reduced with increasing stress ratio.

For simple load histories containing combinations of overload and underload cycles, most available test results suggest that if an underload immediately follows an overload the degree of retardation due to overloading is reduced but not eliminated. An underload applied prior to an overload, on the other hand, has little effect on the degree of crack retardation (Taheri *et al* 2003). Therefore, an underload applied immediately after an overload reduces the post overload retardation more significantly than an underload which immediately precedes an overload, as depicted in Fig. 5.

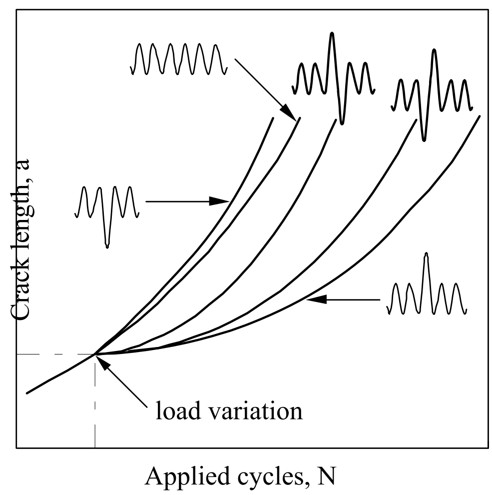

Fig. 5 - Typical crack growth behaviour under several combinations of overload and underload cycles in aluminium alloys.

3 - MODELS FOR VARIABLE AMPLITUDE FATIGUE CRACK GROWTH

Researchers have proposed several models to describe the overload retardation behaviour. Generally, these retardation models may be classified into two main categories: crack tip plasticity models and crack closure models. The crack tip plasticity models are based on that assumption that crack growth retardation occurs due the large plastic zone developed during overloading. The residual compressive stresses formed in this zone will reduce the magnitude of the tensile stresses during the next fatigue cycle and tend to delay the crack growth. The earliest crack tip plasticity models are the Wheeler model (Wheeler 1972) and the Willenborg model (Willemborg 1971), proposed in the early 1970s, and are still used, mainly by their simplicity.

The other main category of retardation models are based on the crack closure approach, considering plastic deformation and crack face interaction in the wake of the crack, proposed by Elber (1972).

3.1 - Wheeler model

The Wheeler model (Wheeler 1972) is a simple model that calculates the fatigue crack growth retardation following a single tensile overload by introducing a retardation parameter, ϕ_R, which prescribes a reduced growth rate for fatigue cracks advancing through the expanded plastic zone produced by the overload.

The retardation parameter ϕ_R can be multiplied to any constant amplitude fatigue model. Therefore, applying the Wheeler model to the Paris law the following equation is obtain:

$$\frac{da}{dN} = \phi_R \left[C \, \Delta K^m \right] \tag{1}$$

The retardation parameter, ϕ_R, is defined as (Wheeler 1972):

$$\phi_R = \begin{cases} \left(\dfrac{r_{p,i}}{a_{OL} + r_{OL} - a_i} \right)^n, & a_i + r_{p,i} < a_{OL} + r_{OL} \\ 1, & a_i + r_{p,i} \geq a_{OL} + r_{OL} \end{cases} \tag{2}$$

where a_{OL} is the crack length when the overload is applied, a_i is the crack length at each load cycle i, r_{OL} is the size of the plastic zone produced by the overload at a_{OL}, $r_{p,i}$ is the size of plastic zone produced by the post-overload constant amplitude loading at current crack length a_i and n is an adjustable experiment-based shaping exponent, which depends on the type of material, geometry and overload magnitude. C and m are those used in the Paris equation.

The size of the plastic zones r_{OL}, and r_P are obtained by the following equations:

$$r_p = \alpha \left(\frac{K_{max}}{\sigma_{YS}} \right)^2 \tag{3.1}$$

$$r_{OL} = \alpha \left(\frac{K_{OL}}{\sigma_{YS}} \right)^2 \tag{3.2}$$

where σ_{YS} is the yield stress of the material and K_{max} is the maximum stress intensity factor at the current crack length a_i. For the constant amplitude loading with a single tensile overload, K_{OL} corresponds to the maximum value of the overload.

The plastic zone size factor α is dependent upon the constraints around the crack tip. In calculating the plastic zone size, several approaches have been proposed. The factor α is $1/\pi$ for plane stress and $1/3\pi$ for plane strain in the Irwin analytical solution (Irwin 1957).

The Wheeler model has had some success in modelling the basic crack growth retardation due to single overloads in an otherwise constant amplitude spectrum. However, this model bears little physical meaning and the experimental determination of the retardation parameter is generally difficult. Moreover, this model cannot deal with the effects of underloads and more difficulties arise when both overloads and underloads are involved.

3.2 - Willemborg model

In the Willemborg model (Willemborg 1971) the amount of retardation following a single overload is obtained using an effective stress intensity factor defined by the following expression:

$$(K_i)_{eff} = K_i - K_{Red} \tag{4}$$

where $(K_i)_{eff}$ and K_i are the effective and apparent (under constant amplitude) stress intensity factors in each load cycle i, respectively.

The modified stress intensity factor that includes retardation, K_{Red}, is given by the following equation:

$$K_{Red} = K_{OL} \sqrt{1 - \frac{a_i - a_{OL}}{r_{OL}}} - K_{max,i} \tag{5}$$

where K_{OL} is the overload stress intensity factor and $K_{max,i}$ the maximum stress intensity factor in each load cycle i.

When the current crack length has extended through the plastic zone produced by the overload the retardation stops, therefore the retardation stress intensity factor is set to zero (K_{Red}=0). Furthermore, any overload with high magnitude than the previous one produces a new retardation effect, which is independent of the previous retardation.

In this model the retardation effect in the crack growth is produced by using the effective stress ratio in each load cycle *i*, $(R_i)_{eff}$, given by,

$$(R_i)_{eff} = \frac{(K_{min,i})_{eff}}{(K_{max,i})_{eff}} \tag{6}$$

in any constant amplitude fatigue crack propagation law which takes into account the stress ratio effect. Willemborg (1972) proposed the Forman equation, therefore,

$$\frac{da}{dN} = \frac{C(\Delta K_{eff})^n}{(1 - R_{eff})K_c - \Delta K_{eff}} \tag{7}$$

where K_c is the critical stress intensity factor and C and n experimental constants obtained under constant amplitude loading.

The Willemborg model does not incorporate any empirical parameters, being only necessary to now the yield stress of the material in order to calculate the plastic zone size. However, this model is not always reliable for predicting overload retardation in some materials (Taheri *et al* 2003), and presents the limitation of predicting crack arrest for overloads with magnitude $K_{OL} \geq 2 \times K_{max,i}$.

The multi-parameter yield zone model, proposed by Johnson (1981), is a modification to the Willenborg model. This modified model is able to account for the crack growth retardation, acceleration and underload effects. However, this improvement of the Willenborg model requires four load interaction parameters. The parameters have to be selected from the simple overload test data, which provide the best fit of the predicted to the experimental test results of these tests.

3.3 - Crack closure model

Based in experimental observations of crack face interaction in the wake of the crack, Elber (1972) argued that a load cycle is only effective in driving the growth of a fatigue crack if the crack tip is fully open, suggesting that the effective stress intensity range, ΔK_{eff}, should be obtain by the following expression:

$$\Delta K_{eff} = K_{max} - K_{op} \tag{8}$$

where K_{op} represents the stress intensity factor corresponding to the opening stress of the crack, σ_{op}. Therefore, the retardation effect in the crack growth can be obtained by,

$$\frac{da}{dN} = f(\Delta K_{eff}) \tag{9}$$

were function *f* can be the Paris law, Forman's equation or other constant amplitude propagation law generally accepted.

The crack closure approach generally permits to correlate the majority of the crack growth transients observed under simple variable amplitude loading sequences using experimental crack closure measurements, namely for peak overloads (Borrego *et al* 2001 and 2003), periodically applied overloads (Borrego *et al* 2005) and two-level block loading (Borrego *et al* 2008). However, the major limitations of this model is that the K_{op} must be obtained for the specific material and loading condition for each load cycle, or by an approximated expression of the evolution of the opening stress proposed from experimental crack closure measurements, which is very complex and time consuming.

Due to the difficulty in determining the opening stress, several models have been developed, which include analytical or numerical procedures, in order to obtain the evolution of the crack closure level under variable amplitude loading for each load cycle.

Among these models one of the most used is the strip yield model proposed by Newman (1981) based on the Dugdale model, but modified to leave plastically deformed material in the wake of the advancing crack tip.

The finite element analysis for computing directly the crack opening stress is complicated and also very time consuming. In the strip yield model proposed by Newman, the crack tip plastic zone and the region of residual plastic deformation is divided up into one-dimensional rigid perfectly plastic elements. The deformation state of these elements is monitored cycle-by-cycle and compared with the elastic displacements of the crack flanks. Contact stresses arise on the crack surfaces in order to meet the compatibility between the elastic crack flanks and the plastically deformed elements. The crack opening stress is then given by the point when the contact stresses become zero.

In addition, the model uses a plastic constraint factor, α, to account for the three-dimensional effects at the crack tip. This constraint factor is used to elevate the flow stress, σ_o, at the crack tip to account for the influence of stress state ($\alpha\sigma_o$) on plastic-zone sizes and crack-surface displacements. The flow stress is taken as the average between the yield stress and ultimate tensile strength of the material. For plane-stress conditions, α is equal to unity (original Dugdale model), and for simulated plane-strain conditions, α is equal to 3.

The equations for computing the opening stress are a complex function of cyclic stress, load history, crack and specimen geometry, constraint and the element density used for calculating the plastic zone size. Therefore, the model is more computationally involved than the previously discussed models.

Moreover, the strip-yield model does not always model the correct yield-zone shape. Therefore the constraint factor is generally used as a curve fitting parameter. Moreover, the effect of the stress history on the magnitude of the constraint factor is not clear and is still subject to ongoing research (ex: Skorupa 2007)

Nonetheless, the strip yield model appears to be able to simulate delay retardation and initial crack growth acceleration immediately following an overload and several spectrum loading sequences.

3.4 –Wheeler model enhancement

The Wheeler retardation model (Wheeler 1972) is one of the simplest and most widely employed models to quantify the fatigue crack growth retardation after a single overload and it offers potential for modifications and improvements.

The model is based on the interaction of the overload plastic zone and the current plastic zone. However, its simple formulation is unable of accounting for the initial crack growth acceleration, delay retardation, the effects of underloads and overload interaction.

Therefore, several authors proposed a number of modifications to the Wheeler model to improve its accuracy in predicting the fatigue crack growth transients following peak overloads, including multiple overloading situations (Yuen and Taheri 2006), as well as block loading (Wang *et al* 2009), and combinations of overloads and underloads, load sequencing and spectrum loadings, taking into account the stress ratio effect (Huang *et al* 2008).

3.4.1 - Crack growth modelling proposed by Yuen and Taheri

Yuen and Taheri (2006) proposed several modifications to the Wheeler model to improve its accuracy in predicting the complete crack growth transients following a single overload, including multiple overloading situations. This model was developed to consider the overload effect on crack growth based on the stress intensity factor. Since the stress intensity factor is an engineering concept for practical applications, the current effort is to search for an existing model that is based on the stress intensity factor.

Yuen and Taheri (2006) incorporated, in the original Wheeler model, two additional parameters, one to account for the delay retardation and the other to take in consideration the overload interaction. Furthermore, they defined an effective stress intensity factor range to account for the initial crack growth acceleration generally observed immediately after overload application (Shin and Hsu 1993; Borrego *et al* 2001).

Similar to the original Wheeler model, Yuen and Taheri (2006) applied there modified Wheeler model to the Paris equation in the following form

$$\frac{da}{dN} = \phi_R \; \phi_D \; \phi_I \left[C \left(\Delta K_{ac} \right)^m \right] \qquad (10)$$

where ϕ_R is the retardation parameter as defined in the original Wheeler model, Eq. (2), ϕ_D is the delay parameter, ϕ_I is the overload interaction parameter and ΔK_{ac} is the accelerated stress intensity factor range. Under constant amplitude stable crack growth, $\phi_R=1$, $\phi_D=1$, $\phi_I=1$ and $\Delta K_{ac}=\Delta K$.

Yuen and Taheri (2006) obtained the retardation parameter ϕ_R calculating the size of the plastic zones r_{OL}, and r_P using Eq. (3.1) and Eq. (3.2), respectively. However, for these authors the α constant of the referred equations is considered the effective plastic zone size constant, which they established by determining from the experimental data the total retardation crack length, *i.e.*, the crack length at which the crack growth rate resumes the constant amplitude crack growth level, a_r.

The delayed-retardation parameter ϕ_D is defined as:

$$\phi_D = \begin{cases} \left(\dfrac{a_{OL} + r_{d,OL} - a_i}{r_{d,i}} \right)^q, & a_i + r_{d,i} < a_{OL} + r_{d,OL} \\ 1, & a_i + r_{d,i} \geq a_{OL} + r_{d,OL} \end{cases} \qquad (11)$$

where $r_{d,OL}$ is the size of the overload effective delay zone, $r_{d,i}$ is the size of the current effective delay zone and q is the shaping exponent for the modified model.

According to Eq. (11) the delay parameter is at its maximum immediately following the overload and decreases to unity at the point of minimum crack growth rate.

The size of the overload effective delay zone and of the current effective delayed zone can be assumed to be calculated in the same manner as the corresponding plastic zones (Eq. (3)):

$$r_d = \beta \left(\frac{K}{\sigma_{YS}} \right)^2 \qquad (12)$$

where β is the effective delay zone size constant, which as to be established by de-

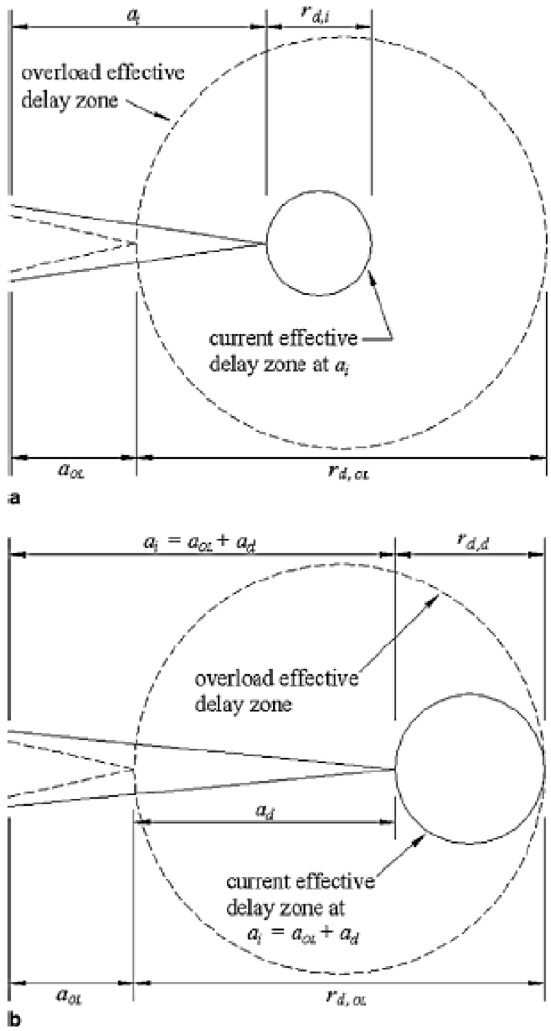

Fig. 6 – Illustration of the terms defining the delay parameter: (a) at a_i; (b) at $a_i=a_{OL}+a_d$; Yuen and Taheri (2006).

termining from the experimental data the crack length at where the crack growth rate achieves the minimum value, a_d.

As illustrated in Fig. 6(a) the overload applied at a_{OL} develops an overload effective delay zone size, $r_{d,OL}$, resulting into the delay crack growth retardation. The delay retardation ceases when the crack propagates further to the crack size $a_i=a_{OL}+a_d$. At this moment the crack tip effective delay zone size is denoted $r_{d,d}$. As illustrated in Fig. 6(b), the relationship of a_d to the effective zone size is expressed by the following expression:

$$a_d = r_{d,OL} - r_{d,d} = \beta \left[\left(\frac{K_{OL}}{\sigma_{YS}} \right)^2 - \left(\frac{K_d}{\sigma_{YS}} \right)^2 \right] \qquad (13)$$

where K_d is the maximum stress intensity at $a_i=a_{OL}+a_d$. Therefore, once the value of a_d is experimentally obtained, the constant β in Eq. (13) is simply determined.

The overload interaction parameter ϕ_I is defined as presented in Eq. (14), where $\phi_{min,i}$ is the minimum value of the retardation parameter ϕ_R, which is calculated from previous overloads at the current crack length.

According to the formulation presented in Eq. (14), during the crack growth, the average value of $\phi_{min,i}$, and thus the overload interaction parameter, decreases and crack growth retardation increases as the spacing between the overloads decreases for the same overload load magnitudes. This is because the number of the overloads from which the minimum retardation parameter is extracted further increases.

The accelerated stress intensity factor range, ΔK_{ac}, is defined in the modified Wheeler model as presented in Eq. (15), where ΔK_{OL} is the overload stress intensity factor range and ΔK_i the current stress intensity factor in each load cycle *i*.

Eq. (15), states that the effect of the overload stress intensity factor range on crack growth acceleration is at its maximum immediately following the overload and decreases to zero at the end of the delay retardation crack length.

In the work of Yuen and Taheri (2006), both single and multiple overloading fatigue tests were performed on 350 WT steel specimens and the fatigue life predictions by the original and the modified Wheeler model were compared. Contrary to the original model, the modified Wheeler model was able to predict the initial acceleration and the delay retardation under single peak overloads. Moreover, equations (10) to (15) were able to reasonably describe the crack growth transients under multiple overloading.

Wang *et al* (2009) also applied the modified Wheeler model proposed by Yuen and Taheri (2006), setting the overload interaction parameter $\phi_I=1$, for analysing single tensile overloads as well as two-step high-low loading sequences on 16MnR steel specimens. The stress intensity factor range ΔK_{OL} was set to be $K_{OL}-K_{min}$ for the overloading condition, where K_{min} corresponds to the minimum load of the constant amplitude loading. For the high-low sequence loading, ΔK_{OL} was set to be the stress intensity factor range ΔK_{Hi} of the preceding higher loading step. It was clear that the modified Wheeler model can predict reasonably the overall crack growth transients produced by single tensile overloads and high-low block loading sequences.

However, the modified Wheeler model proposed by Yuen and Taheri (2006) contains three additional constants, namely, α, q and β, which need to be experimentally obtained. Furthermore, similar to the original model, the modified model does not consider the stress ratio effect.

$$\phi_I = \begin{cases} 1-\left(1-\phi_{min,i}\right)\left[1-\dfrac{r_{p,i}}{a_{OL}+r_{p,OL}-a_i}\dfrac{a_{OL}+r_{d,OL}-a_i}{r_{d,i}}\right]^q, & a_i+r_{d,i}<a_{OL}+r_{d,OL} \\ 1-\left(1-\phi_{min,i}\right)\left[1-\dfrac{r_{p,i}}{a_{OL}+r_{p,OL}-a_i}\right]^q, & a_{OL}+r_{d,OL}\le a_i+r_{d,i}\,,\, a_i+r_{p,i}<a_{OL}+r_{p,OL} \\ 1, & a_{OL}+r_{p,OL}\le a_i+r_{p,i} \end{cases} \tag{14}$$

$$\Delta K_{ac} = \begin{cases} \Delta K_i+\left(\Delta K_{OL}-\Delta K_i\right)\left[1-\dfrac{r_{d,i}}{a_{OL}+r_{d,OL}-a_i}\right]^q, & a_i+r_{d,i}<a_{OL}+r_{d,OL} \\ \Delta K_i, & a_{OL}+r_{p,OL}\le a_i+r_{p,i} \end{cases} \tag{15}$$

Therefore, additionally to the experiment-based shaping exponent of the original model, n, the values of the effective plastic size constant, α, the shaping exponent, q, and the fitting constant, β, have to be experimental determined for each combination of R ratio and overload level, K_{OL}, from several single overload fatigue tests.

3.4.2 - Crack growth modelling proposed by Xiaoping Huang et al

It is well known that crack growth rates expressed in terms of the stress intensity factor range, ΔK, depend on the stress ratio, R. Based on Kujawski (2001) two parameter driving force model, Huang and Moan (2007) proposed an equivalent stress intensity factor range model, which condenses the data obtained for different stress ratios under constant amplitude loading into a single curve scaled to R=0.

Afterwards, Huang *et al* (2008) extended this model to take into account the loading sequence effect on crack growth under variable amplitude loading. They introduced the retardation parameter ϕ_R of the original Wheeler model, with small modification to consider underload effects, into their equivalent stress intensity factor range model (Huang and Moan 2007), covering the stress ratio effect in stages I and II of the crack growth rate, which lead to the following expressions:

$$\frac{da}{dN} = C\left[\left(\Delta K_{eq0}\right)^m - \left(\Delta K_{th0}\right)^m\right] \qquad (16)$$

$$\Delta K_{eq0} = M_R M_P \Delta K \qquad (17)$$

where ΔK_{th0} is the threshold at R=0, C and m are those corresponding to R=0 and parameters M_R and M_P are correction factors for the equivalent stress intensity factor range.

M_R is the correction factor of the stress ratio effect, *i.e.*, the parameter that promotes a simply horizontal shift of the curves corresponding do different R ratios to the curve of R=0, and M_P is the correction factor to model the load interaction effects, which replaces the retardation parameter ϕ_R of the original Wheeler model. The R ratio correction factor M_R is defined as:

$$M_R = \begin{cases} (1-R)^{-\beta_1}, & -5 \le R \le 0 \\ (1-R)^{-\beta}, & 0 \le R \le 0.5 \\ \left(1.05-1.4R+0.6R^2\right)^{-\beta}, & 0.5 \le R \le 1 \end{cases} \qquad (18)$$

where β and β_1 are parameters depending on material properties ($\beta \le \beta_1 \le 1$).

The present model implies that the crack growth rate exponent m is assumed to be independent of the R ratio. Parameters β and β_1 are set to certain values for the same kind of materials, for example, for aluminium alloys and steels β=0.7 and β_1=1.2 β (Huang and Moan 2007).

The loading sequence interaction correction factor M_P is defined as:

$$M_P = \begin{cases} \left(\dfrac{r_P}{a_{OL}+r_{OL}-a-r_\Delta}\right)^n, & a+r_P < a_{OL}+r_{OL}-r_\Delta \\ 1, & a+r_P \ge a_{OL}+r_{OL}-r_\Delta \end{cases} \qquad (19)$$

where n is a shaping exponent determined by fitting to the experimental data, set to the same value for the same material. The other variables are depicted in Fig. (7), where r_y represents r_p in Eq. (19).

Since an underload reverses the plastic flow and depletes the resulting plastic zone, the yield zone size reduction caused by an underload can be quantitatively calculated using the following equations:

$$r_\Delta = \alpha\left(\frac{\Delta K_u}{\bar{\sigma}_{YS}}\right)^2 \qquad (20)$$

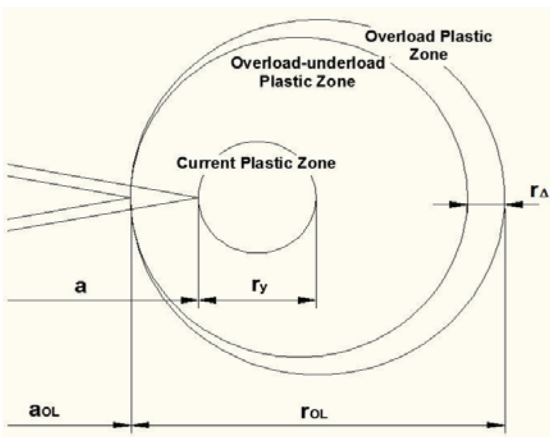

Fig. 7 – Illustration showing the variables and zones associated with the model proposed by Huang *et al* (2008),

$$\Delta K_u = Y\sqrt{\pi a}\left(\sigma_{min}^{i-1} - \sigma_{min}^{i}\right) \qquad (21)$$

where σ_{YS}^{-} is the compressive yield stress and $\sigma_{min}^{i-1} - \sigma_{min}^{i}$ the minimum stresses up to and following the *i*th overload, respectively. The size of the plastic zone in front of the crack tip is dependent upon the constraint state around the crack tip, which is a function of the maximum stress, yield strength and plate thickness. In this model the plastic size factor α is modelled as a continuous function of these variables (Huang *et al* 2008), making the calculation of the plastic zone sizes easy and more precise than those proposed by other researchers (Voorwald *et al* 1991; Guo 1999).

In the work of Huang *et al* (2008), fatigue life perditions using this modified Wheeler model were performed in several materials under different variable amplitude loading sequences, namely: single, multiple and periodically applied overloads on 7075-T6 aluminium alloy; underload following an overload and vice versa, as well as loading spectrum with steps in *R* ratio on 2024-T3 aluminium alloy and multiple overloads (2 or 3) on 350 WT steel. The comparisons between experimental and predicted results showed that the model's accuracy is satisfactory.

However, it is important to notice that Huang *et al* (2008) represented, in all variable amplitude loading sequences analysed, the experimental data and the corresponding predictions only as *a-N* curves. Therefore, namely for single overloads, these researchers did not verified if this model is able to predict the initial acceleration and the following delayed retardation phases, only observed in the *da/dN-ΔK* curves, following overload application.

4- CONCLUSIONS

In this paper several concepts, trends and prediction models of crack growth under variable amplitude loading have been presented and evaluated.

The details of the analysed fatigue crack growth models under variable amplitude loading have been presented highlighting the merits and limitations of each model. From the literature it is observed that most of the models require one or more calibration parameters or constants to conduct crack growth analysis. Therefore, it is clear that curve fitting is used as an imported procedure to correlate the predictions with the experimental data. Furthermore, the predictions are strongly influenced by the parameters which have to be fitted to experimental data.

Due to the number and complexity of the mechanisms involved in this problem, no universal model exists yet. The selection of the appropriate model is usually based on the researcher experience and persona1 preference, thus accurate predictions remain problematic. Therefore, there is considerable scope to improve the present models and development of better and simpler ones.

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