MODAL SENSING OF BEAMS VIA SPATIALLY SHAPED DISTRIBUTED PIEZOELECTRIC TRANSDUCERS

C. M. A. Vasques, J. Dias Rodrigues
Departamento de Engenharia Mecânica e Gestão Industrial,
Faculdade de Engenharia da Universidade do Porto,
Rua Dr. Roberto Frias s/n, 4200-465 Porto, Portugal
E-mail: cvasques@fe.up.pt

ABSTRACT
In this article, modal sensing via spatially shaped distributed piezoelectric transducers is investigated for beams. A simple beam model considering the electromechanical coupling effects is presented and the spatially distribution of modal sensors is discussed.

1. INTRODUCTION
The concept of using spatially shaped distributed piezoelectric transducers in order to filter out undesirable mode’s contributions, making them unobservable to the shaped sensor’s voltage over the bandwidth of interest, in the field of active vibration (AVC) and/or structural acoustic control (ASAC), has been extensively investigated since the 1980s [Lee et al. (1991)]. Some of the most representative studies include modal sensors [Lee and Moon (1990)] and spatial filtering [Collins et al. (1994)] technologies.

In this article, modal sensing via spatially shaped distributed piezoelectric transducers is investigated for beams. With that purpose, a simple beam model considering the electromechanical coupling effects is presented and the spatially distribution of modal sensors is discussed. A case study of a clamped beam is considered and the modal sensing performance is assessed by means of sensing voltage to transverse force loading frequency response functions (FRFs).

2. MECHANICAL MODEL OF THE BEAM
Following the well known Euler-Bernoulli assumptions for thin beams, the transverse vibration of beams with a constant cross sectional area is governed by [Timoshenko et al. (1974)]

\[ E_b I_b \frac{\partial^4 w(x,t)}{\partial x^4} + \rho_b A_b \frac{\partial^2 w(x,t)}{\partial t^2} = f(x,t) \]

where \( x \) and \( t \) are the beam’s spatial and time coordinates, \( w(x,t) \) is the beam’s transverse displacement, \( f(x,t) \) is a distributed transverse force, \( E_b \) is the Young’s modulus, \( A_b \) is the cross sectional area, \( I_b \) is the second-order moment of area and \( \rho_b \) the mass density and the subscript \( b \) is used to denote beam quantities.

The solution of the governing equation of motion in Eq. (1) might be expressed as a linear combination of the mode shapes,

\[ w(x,t) = \sum_{\eta=1}^{\infty} \varphi\eta(x) \eta\eta(t) \]
where \( \phi_r(x) \) and \( \eta_r(t) \) are the \( r \)th mode shape and modal coordinate, respectively.

### 3. PIEZOELECTRIC SENSING

For piezoelectric materials of the crystal class \( mm2 \) polarized in the transverse direction, the one-dimensional sensing behavior is given by

\[
D_3 = e_{3i} e_{ii} - e_{33}^r E_i, \tag{3}
\]

where \( e_{3i} \) is the piezoelectric stress constant, \( e_{33}^r \) is the dielectric constant under constant stress and \( D_3 \) and \( E_i \) are the transverse electric displacement and electric field, respectively.

According to the Euler-Bernoulli assumptions, the extensional strain component is given by

\[
\epsilon_{ii}(x, z, t) = -z \frac{\partial^2 w(x, t)}{\partial x^2}, \tag{4}
\]

where \( z \) is the spatial coordinate in the transverse direction, starting at the beam neutral axis.

If the electrodes are short-circuited, the enforced electric field will be zero, i.e., \( E_i = 0 \). Thus, considering Eq. (3), the transverse electrical displacement is related with the enforced strain by

\[
D_3 = e_{3i} e_{ii} \eta_r(t), \tag{8}
\]

Thus, considering the spatial sensitivity function \( S(x) \) proportional to the generic \( s \)th modal strain distribution along the length of the beam, i.e.,

\[
S(x) = \alpha_s S(x) = \alpha_s \frac{E_i l_s}{\omega_i^2} \frac{d^2 \phi_s(x)}{dx^2}, \tag{9}
\]

where \( \alpha_s \) is a normalization factor, and substituting Eq. (9) into (8), yields

\[
v(t) = -\frac{e_{3i} h_2}{C_p} \sum_{r=1}^{\infty} \left( \int_{x} S(x) \frac{d^2 \phi_r(x)}{dx^2} dx \right) \eta_r(t). \tag{10}
\]

According to the orthonormality properties of the modal functions, Eq. (10) reduces to

\[
v(t) = -\frac{e_{3i} h_2}{C_p} \alpha_s \eta_s(t), \tag{11}
\]

which is to say that the sensing voltage is only proportional to the \( s \)th mode contribution to the net vibratory response of the beam, thus acting as a modal sensor filter tuned only to the \( s \)th mode, filtering all the other mode’s contributions.
In general, if we want to sense more than one mode shape, i.e., to define a specific bandwidth over which the modal sensor should work or specific modes to be sensed, Eq. (9) can be generalized to

\[ S(x) = \beta \sum_{k=1}^{l} S_k(x) = \beta \sum_{k=1}^{l} \left( \frac{E_t l_k}{\omega_k^3} \frac{d^2 \phi_k(x)}{dx^2} \right), \tag{12} \]

where \( l \) is the upper mode number of the bandwidth of interest, \( m \) are the mode numbers of the modes that we wish to make unobservable and \( \beta \) is another multimode normalization factor. Thus, Eq. (11) can also be generalized to

\[ v(t) = -\frac{\epsilon_s \cdot h \cdot 2b}{\rho \cdot C_p} \beta \sum_{k=1}^{l} \alpha_k \eta_k(t). \tag{13} \]

This can be seen as an interesting strategy to alleviate the spillover effects by restring the observability of the sensor to the specific modes or bandwidth of interest by tailoring the electrode profile according to Eq. (12).

5. FREQUENCY RESPONSE FUNCTION

Considering an harmonic concentrated transverse force disturbance of amplitude \( F \) located at \( x = x_f \), so that

\[ f(x,t) = F \delta(x-x_f) e^{j \omega t}, \tag{14} \]

the sensing voltage to force FRF, considering the modal filtering approach in Eq. (13), is given by

\[ V(j \omega) = \frac{F}{-\frac{\epsilon_s \cdot h \cdot 2b}{\rho \cdot C_p} \beta \sum_{k=1}^{l} \alpha_k \phi_k(x_f)} \left( \omega_k^2 - \omega^2 \right) + j2 \xi \omega \omega \tag{15} \]

6. CASE STUDY

In order to illustrate the modal filtering approach a case study comprising a cantilever beam with a spatially shaped distributed modal piezoelectric sensing patch along the length of the beam is considered. The sensor is 50 mm thick and the beam’s dimensions are 20 × 3 × 400 mm. The aluminum beam and piezoelectric patch (a generic PVDF) material properties are given in Tab. 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>( E_s ) [Pa]</td>
<td>70 × 10^9</td>
</tr>
<tr>
<td>PVDF</td>
<td>( e_{33} ) [C m^-2]</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>( \rho_b ) [Kg m^-3]</td>
<td>2700</td>
</tr>
<tr>
<td></td>
<td>( \epsilon_{33} ) [F m^-1]</td>
<td>100 × 10^-12</td>
</tr>
</tbody>
</table>

In order to demonstrate the effectiveness of the modal sensors to turn specific modes unobservable according with the shape of the sensor, two situations were considered. In the first one, the sensor was tailored so that only the 1st flexural mode of vibration is observed. In the second case the design purpose was to make all but the 2nd and 4th modes unobservable. The correspondent single and multimode sensor shapes and the correspondent sensing voltage to force FRFs to a transverse force applied at the free edge of the beam are presented in Figs. 2 and 4 and Figs. 3 and 5, respectively. Moreover, the results are also compared with the ones obtained with a uniformly distributed piezoelectric sensor.

As can be seen, when compared with the uniform sensor, the single and multimode sensors manage to filter out the unwanted modal contributions.

![Fig 2: Shape of the tailored piezoelectric single mode sensor tuned to the 1st mode.](image-url)
7. CONCLUSION

In this article modal sensors via spatially shaped distributed piezoelectric transducing technologies were investigated and successfully shown to meet the design purposes. A simple beam model considering the electromechanical coupling effects was presented and the spatially distribution of modal sensors was discussed and assessed.

It was shown that modal sensors can effectively be used to reduce spillover effects due to unobserved model dynamics avoiding, in some extent, the use of electronic or digital filtering in vibration control applications. Furthermore, another interesting feature observed is that the induced electrical signal for modal sensors was shown to be, in general, higher than the one obtained with uniform sensors. Therefore, this feature can be very interesting for dissipation or damping purposes using shunted piezoelectric transducers since the electromechanical coupling is higher for modal sensors therefore allowing more mechanical to electrical energy conversion and energy (electrical) to be dissipated.

REFERENCES


