MODAL SENSING OF BEAMS VIA SPATIALLY SHAPED DISTRIBUTED PIEZOELECTRIC TRANSDUCERS

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ABSTRACT

In this article, modal sensing via spatially shaped distributed piezoelectric transducers is investigated for beams. A simple beam model considering the electromechanical coupling effects is presented and the spatially distribution of modal sensors is discussed and assessed.

1. INTRODUCTION

The concept of using spatially shaped distributed piezoelectric transducers in order to filter out undesirable mode's contributions, making them unobservable to the shaped sensor's voltage over the bandwidth of interest, in the field of active vibration (AVC) and/or structural acoustic control (ASAC), has been extensively investigated since the 1980s [Lee *et al.* (1991)]. Some of the most representative studies include modal sensors [Lee and Moon (1990)] and spatial filtering [Collins *et al.* (1994)] technologies.

In this article, modal sensing via spatially shaped distributed piezoelectric transducers is investigated for beams. With that purpose, a simple beam model considering the electromechanical coupling effects is presented and the spatially distribution of modal sensors is discussed. A case study of a clamped beam is considered and the modal sensing performance is assessed by means of sensing voltage to transverse force loading frequency response functions (FRFs).

2. MECHANICAL MODEL OF THE BEAM

Following the well known Euler-Bernoulli assumptions for thin beams, the transverse vibration of beams with a constant cross sectional area is governed by [Timoshenko *et al.* (1974)]

$$E_b I_b \frac{\partial^4 w(x,t)}{\partial x^4} + \rho_b A_b \frac{\partial^2 w(x,t)}{\partial t^2} = f(x,t)\,, \qquad (1)$$

where x and t are the beam's spatial and time coordinates, $w(x,t)$ is the beam's transverse displacement, $f(x,t)$ is a distributed transverse force, E_b is the Young's modulus, A_b is the cross sectional area, I_b is the second-order moment of area and ρ_b the mass density and the subscript b is used to denote beam quantities.

The solution of the governing equation of motion in Eq. (1) might be expressed as a linear combination of the mode shapes,

$$w(x,t) = \sum_{r=1}^{\infty} \varphi_r(x)\eta_r(t)\,, \qquad (2)$$

where $\varphi_r(x)$ and $\eta_r(t)$ are the rth mode shape and modal coordinate, respectively.

3. PIEZOELECTRIC SENSING

For piezoelectric materials of the crystal class mm2 polarized in the transverse direction, the one-dimensional sensing behavior is given by

$$D_3 = e_{31}\varepsilon_{11} - \epsilon_{33}^{T} E_3 , \tag{3}$$

where e_{31} is the piezoelectric stress constant, ϵ_{33}^{T} is the dielectric constant under constant stress and D_3 and E_3 are the transverse electric displacement and electric field, respectively.

According to the Euler-Bernoulli assumptions, the extensional strain component is given by

$$\varepsilon_{11}(x,z,t) = -z\frac{\partial^2 w(x,t)}{\partial x^2}, \tag{4}$$

where z is the spatial coordinate in the transverse direction, starting at the beam neutral axis.

If the electrodes are short-circuited, the enforced electric field will be zero, i.e., $E_3 = 0$. Thus, considering Eq. (3), the transverse electrical displacement is related with the enforced strain by $D_3 = e_{31}\varepsilon_{11}$. Integrating the electrical displacement over the electrode/piezoelectric patch area A_e, and considering the mechanical strain definition in Eq. (4), the induced electric charge $q(t)$, due to the enforced mechanical strain, is given by

$$\begin{aligned} q(t) &= -\int_{A_e} e_{31}h_b \frac{\partial^2 w(x,t)}{\partial x^2}\mathrm{d}A_e \\ &= -e_{31}h_b\, 2b_b \int_{L_p} S(x)\frac{\partial^2 w(x,t)}{\partial x^2}\mathrm{d}x , \end{aligned} \tag{5}$$

where $2h_b$, $2b_b$ and L_p denote the beam's thickness, width and length, respectively, and $S(x)$ is a spatial sensitivity function of an arbitrary shaped piezoelectric sensor, comprised in the interval $[-1,1]$, so that $\mathrm{d}A_e = 2b_b S(x)\mathrm{d}x$, which represents the effective area of the electrode/piezoelectric patch over which the electric displacement is integrated. The sensing voltage $v(t)$ is then proportional to the electric charge,

$$v(t) = \frac{q(t)}{C_p} = -\frac{e_{31}h_b 2b_b}{C_p}\int_{L_p} S(x)\frac{\partial^2 w(x,t)}{\partial x^2}\mathrm{d}x , \tag{6}$$

where C_p is the piezoelectric patch capacitance, defined in terms of the effective electrode area so that $\mathrm{d}A_{\text{eff}} = 2b_b\left|S(x)\right|\mathrm{d}x$, given by

$$C_p = \frac{\epsilon_{33}^{T} A_{\text{eff}}}{2h_p} . \tag{7}$$

According to Eq. (6), the sensing voltage is defined in terms of the beam's curvature and depends of the chosen spatial sensitivity function.

4. SPATIAL MODAL FILTERING

Consider the beam with an arbitrary spatially shaped distributed piezoelectric sensor as depicted in Fig. 1. Substituting the modal expansion of the transverse displacement defined in Eq. (2) into Eq. (6), yields

$$v(t) = -\frac{e_{31}h_b 2b_b}{C_p}\sum_{r=1}^{\infty}\left(\int_{L_p} S(x)\frac{\mathrm{d}^2\varphi_r(x)}{\mathrm{d}x^2}\mathrm{d}x\right)\eta_r(t). \tag{8}$$

Thus, considering the spatial sensitivity function $S(x)$ proportional to the generic sth modal strain distribution along the length of the beam, i.e.,

$$S(x) = \alpha_s S_s(x) = \alpha_s \frac{E_b I_b}{\omega_s^2}\frac{\mathrm{d}^2\varphi_s(x)}{\mathrm{d}x^2}, \tag{9}$$

where α_s is a normalization factor, and substituting Eq. (9) into (8), yields

$$\begin{aligned} v(t) &= -\frac{e_{31}h_b 2b_b}{C_p} \\ &\sum_{r=1}^{\infty}\left(\int_{L_p}\alpha_s \frac{E_b I_b}{\omega_s^2}\frac{\mathrm{d}^2\varphi_s(x)}{\mathrm{d}x^2}\frac{\mathrm{d}^2\varphi_r(x)}{\mathrm{d}x^2}\mathrm{d}x\right)\eta_r(t). \end{aligned} \tag{10}$$

According to the orthonormality properties of the modal functions, Eq. (10) reduces to

$$v(t) = -\frac{e_{31}h_b 2b_b}{C_p}\alpha_s\,\eta_s(t) , \tag{11}$$

which is to say that the sensing voltage is only proportional to the sth mode contribution to the net vibratory response of the beam, thus acting as a modal sensor filter tuned only to the sth mode, filtering all the other mode's contributions.

In general, if we want to sense more than one mode shape, i.e., to define a specific bandwidth over which the modal sensor

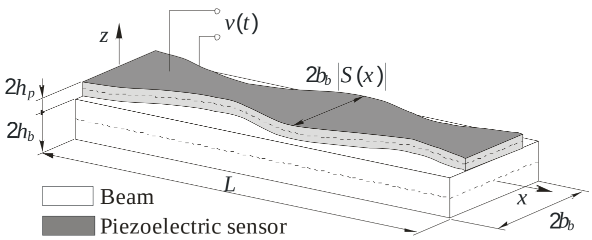

Fig 1: Generic beam structure with an arbitrary spatially shaped distributed piezoelectric sensor.

should work or specific modes to be sensed, Eq. (9) can be generalized to

$$S(x)=\beta\sum_{\substack{k=1\\k\neq m}}^{l}S_k(x)=\beta\left(\sum_{\substack{k=1\\k\neq m}}^{l}\alpha_k\frac{E_bI_b}{\omega_k^2}\frac{\mathrm{d}^2\varphi_k(x)}{\mathrm{d}x^2}\right), \quad (12)$$

where l is the upper mode number of the bandwidth of interest, m are the mode numbers of the modes that we wish to make unobservable and β is another multimode normalization factor. Thus, Eq. (11) can also be generalized to

$$v(t)=-\frac{e_{31}h_b2b_b}{C_p}\beta\sum_{\substack{k=1\\k\neq m}}^{l}\alpha_k\eta_k(t)\,. \quad (13)$$

This can be seen as an interesting strategy to alleviate the spillover effects by restringing the observability of the sensor to the specific modes or bandwidth of interest by tailoring the electrode profile according to Eq. (12).

5. FREQUENCY RESPONSE FUNCTION

Considering an harmonic concentrated transverse force disturbance of amplitude F located at $x=x_F$, so that

$$f(x,t)=F\delta(x-x_F)\mathrm{e}^{\mathrm{j}\omega t}, \quad (14)$$

the sensing voltage to force FRF, considering the modal filtering approach in Eq. (13), is given by

$$\frac{V(\mathrm{j}\omega)}{F}=-\frac{e_{31}h_b2b_b}{C_p}\beta\sum_{\substack{k=1\\k\neq m}}^{l}\alpha_k\frac{\varphi_k(x_F)}{(\omega_k^2-\omega^2)+\mathrm{j}(2\xi_k\omega_k\omega)}. \quad (15)$$

6. CASE STUDY

In order to illustrate the modal filtering approach a case study comprising a cantilever beam with a spatially shaped distributed modal piezoelectric sensing patch along the length of the beam is considered. The sensor is 50 mm thick and the beam's dimensions are $20\times3\times400$ mm. The aluminum beam and piezoelectric patch (a generic PVDF) material properties are given in Tab. 1.

Table 1: Material properties of the beam and PVDF.

Aluminum		PVDF	
E_b[Pa]	70×10^9	e_{31}[C m^{-2}]	0.025
ρ_b[Kg m^{-3}]	2700	$\in_{33}^T$ [F m^{-1}]	100×10^{-12}

In order to demonstrate the effectiveness of the modal sensors to turn specific modes unobservable according with the shape of the sensor, two situations were considered. In the first one, the sensor was tailored so that only the 1st flexural mode of vibration is observed. In the second case the design purpose was to make all but the 2nd and 4th modes unobservable. The correspondent single and multimode sensor shapes and the correspondent sensing voltage to force FRFs to a transverse force applied at the free edge of the beam are presented in Figs. 2 and 4 and Figs. 3 and 5, respectively. Moreover, the results are also compared with the ones obtained with a uniformly distributed piezoelectric sensor.

As can be seen, when compared with the uniform sensor, the single and multimode sensors manage to filter out the unwanted modal contributions.

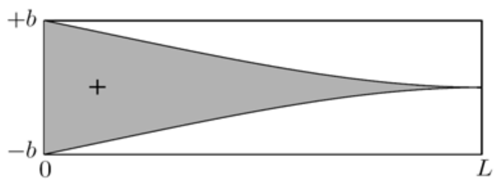

Fig 2: Shape of the tailored piezoelectric single mode sensor tuned to the 1st mode.

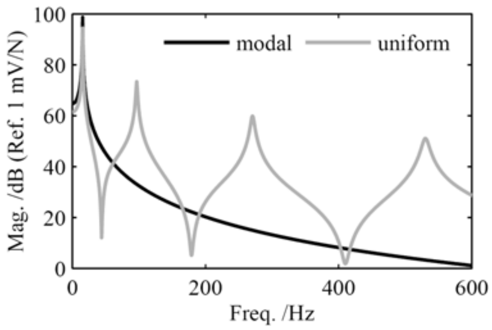

Fig 3: Sensing voltage to force FRF of the modal (1st mode) and uniform sensors.

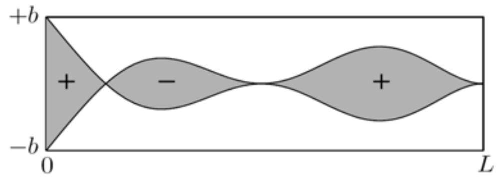

Fig 4: Shape of the tailored piezoelectric multimode sensor tuned to the 2nd and 4th modes.

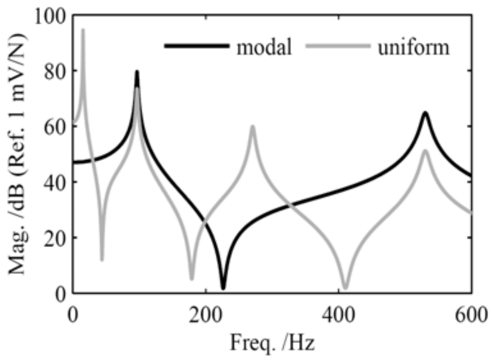

Fig 5: Sensing voltage to force FRF of the modal (2nd and 4th modes) and uniform sensors.

7. CONCLUSION

In this article modal sensors via spatially shaped distributed piezoelectric transducing technologies were investigated and successfully shown to meet the design purposes. A simple beam model considering the electromechanical coupling effects was presented and the spatially distribution of modal sensors was discussed and assessed.

It was shown that modal sensors can effectively be used to reduce spillover effects due to unobserved model dynamics avoiding, in some extent, the use of electronic or digital filtering in vibration control applications. Furthermore, another interesting feature observed is that the induced electrical signal for modal sensors was shown to be, in general, higher than the one obtained with uniform sensors. Therefore, this feature can be very interesting for dissipation or damping purposes using shunted piezoelectric transducers since the electromechanical coupling is higher for modal sensors therefore allowing more mechanical to electrical energy conversion and energy (electrical) to be dissipated.

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