JOURNAL BEARINGS SUBJECTED TO DYNAMIC LOADS: THE ANALYTICAL MOBILITY METHOD

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ABSTRACT
The main purpose of this work is to use the analytical mobility method to analyze journal bearings subjected to dynamic loads, with the intent to include it in a general computational program that has been developed for the dynamic analysis of general mechanical systems. A simple journal bearing subjected to a dynamic load is chosen as a demonstrative example, in order to provide the necessary results for a comprehensive discussion of the methodology presented.

1. INTRODUCTION

Most mechanical systems are expected to work in different regimes. In what respects to the journal bearing, as part of a equipment, this means that variations of rotating speed and of magnitude and direction of the applied load must be accounted for in the design stage. Examples include reciprocating machinery such as compressors, internal combustion engines and other industrial processing machinery.

Lubricated joints are designed so that, even in the worst conditions, journal and bearing are not expected to come into contact. Two very good reasons for designing journal bearings in this way are to reduce friction and to minimize wear and rupture risks. Furthermore, the hydrodynamic fluid film developed plays an important role in the stability of the mechanical systems, due to its damping characteristics, which can not be disregarded.

Lubrication theory for the dynamically loaded journal bearing is mathematically complex and, over the last few decades, several analytical approaches have been proposed. The multigrid techniques based on the Elrod algorithm [1] and the finite element methods [2] of analysis are among the most popular. The finite element methods are probably the most accurate and versatile, but tend to be very time consuming and require high level of knowledge, not accessible to the common designer and, so, remaining confined to research and development. Therefore, based on simplifying premises, engineers and designers prefer to use simpler and still accurate methods, such as the mobility method [3,4] and the impedance method [5,6]. In general, these two approximate techniques, which belong to the category of rapid methods, are employed to perform analysis of simple journal bearings.

The modelization of various types of joints in mechanical systems has recently received considerable attention from several authors [7-9]. The concept of perfect joint is often used in simulating the dynamic of mechanical systems. Although, effects due to clearance, friction and lubrication are all the time in the real joints, which making the perfect joint concept unreal. Therefore, over the last few years, a number of research works have been developed to incorporate these phenomena. The first models only included the effect of clearance by modelling the joint elements as colliding bodies, being the dynamics of the joints controlled by contact forces [7]. This dry model was later improved in order to incorporate the friction effect and expanded to three-dimensional mechanical systems [9]. Ravn et al. [10], based on the work of Pinkus and Sternlicht [11], developed a simple model for dynamic analysis of mechanical systems with lubricated joints. This model, in spite of accounting for the squeeze-film and wedge-film effects in dynamically loaded journal bearings, is only valid for long journal bearings. Schwab et al. [12] used the impedance method to study the influence of the fluid lubricant on the dynamics of a slider-crank mechanism with a clearance joint. The main advantage of this model is that it takes into account the length of the journal bearing, that is, it considered the fact of the journal being shorter or longer. However, the model, based on the impedance method, is more adequate for rotor dynamics than for study the joints in mechanical systems like the one that is found in internal combustion engines [13]. This circumstance is due to the fact that journal and crank masses are relatively low compared to the applied loads on the journal bearings. While, the journal bearing masses in the rotor dynamics are relative large and can influence their dynamic. Flores et al. [14] proposed a new methodology that represents the dry clearance joints and the effect of the lubrication, including a transition phase between the dry contact and lubricated models.

This work aims to use the analytical mobility method to study journal bearings subjected to dynamic loads, with the intent to include it in a general computational program, such as the DAP (Dynamic Analysis Program), that has been developed for the dynamic analysis of general mechanical systems. A simple journal bearing subjected to a dynamic load is chosen as a demonstrative example, in order to provide the necessary results for the comprehensive discussion of the model presented throughout this work.

2. GEOMETRIC CHARACTERISTICS OF JOURNAL BEARINGS

2.1. Journal Bearing Geometry

When the journal and bearing have relative rotational velocities with respect to each other, the amount of eccentricity adjusts itself until the pressure generated in the converging lubricating film balances the external loads. Figure 1 depicts the basic geometric of a journal bearing. The pressure generated, and, hence, the load capacity of the journal bearing, depends on the journal eccentricity, the relative angular velocity, the effective viscosity of the fluid lubricant and the journal bearing geometry and clearance. Usually, the effect of hydrodynamic pressure is looked as the contribution of two different actions; wedge and squeezing. The squeeze action relates the radial journal motion with the generation of load carrying capacity in the lubricant film, whilst the wedge action deals with the relation between relative rotational velocity of the journal and bearing ability to produce such pressure.

The film thickness can be written as,

$$h = c(1 + \varepsilon cos\theta) \tag{1}$$

where c is the radial clearance of the journal bearing, ε is the eccentricity ratio and θ represents the coordinate in the circumferential direction, being measured from the maximum film thickness.

The components of vector eccentricity in the X and Y (Fig.1) directions are expressed by,

$$e_x = e\, sin\, \phi \tag{2}$$

$$e_y = -e\, cos\, \phi \tag{3}$$

or in non-dimensional form,

$$\varepsilon_x = \varepsilon\, sin\, \phi \tag{4}$$

$$\varepsilon_y = -\varepsilon\, cos\, \phi \tag{5}$$

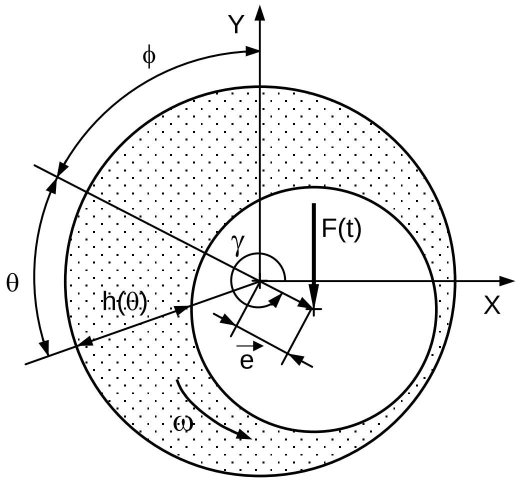

Fig.1 – Basic geometry of a journal bearing.

2.2. Journal Velocities

The relative motion of the journal and the bearing surfaces introduces both tangential, U, and normal, V, components of velocity. These velocities can be expressed in terms of the journal bearing components by noting that the journal rotates about its own axis and translates with respect to the bearing centre. Figure 2 shows the velocity components in a cross section of a journal bearing dynamically loaded. A point P on the journal surface is located at an angle θ from the line of centres, whereas the tangential velocity U takes into account the rotational and translational components of motion.

The tangential velocity component on the journal surface, due to the journal rotation, is equal to ωR, while the translational velocity components of the journal with respect to the bearing centre are $\dot{e}$ and $e\dot{\phi}$ as illustrated in Fig.2. The tangential velocity component at point P, due to the journal rotation, is $\omega R cos\alpha$. Since the angle α is very small, $\omega R cos\alpha$ is approximately equal to ωR. In addition, the tangential velocity component at point P caused by the translational motion is $\dot{e}\, sin\,\theta - e\dot{\phi}\, cos\,\theta$. Thus, the tangential velocity at point P can be written as,

$$U = \omega R + \dot{e}\, sin\,\theta - e\dot{\phi}\, cos\,\theta \tag{6}$$

or

$$U = \omega R + c\dot{\varepsilon}\, sin\,\theta - c\varepsilon\dot{\phi}\, cos\,\theta \tag{7}$$

In a similar way, the normal velocity, V, is equal to sum of the normal components of the rotational and translational velocities. The normal velocity component due to rotational velocity is $\omega R sin\alpha$. Again, noting that α is very small, then $\omega R sin\alpha$ is approximately equal to $\omega R tan\alpha = \omega \partial h / \partial\theta$ [15]. The translational motion of the journal also causes normal velocity component as $\dot{e}\, cos\,\theta + e\dot{\phi}\, sin\,\theta$. Thus, the global normal velocity at point P is expressed by,

$$V = \omega \frac{\partial h}{\partial\theta} + \dot{e}\, cos\,\theta + e\dot{\phi}\, sin\,\theta \tag{8}$$

or

$$V = \omega \frac{\partial h}{\partial\theta} + c\dot{\varepsilon}\, cos\,\theta + c\varepsilon\dot{\phi}\, sin\,\theta \tag{9}$$

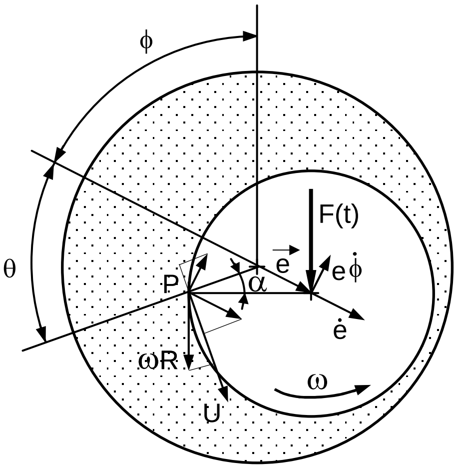

Fig.2 – Cross section of a journal bearing dynamically loaded: velocity components.

3. REYNOLDS' EQUATION FOR DYNAMICALLY LOADED JOURNAL BEARINGS

3.1. Pressure Distribution

The Reynolds' equation for a journal bearing subjected to dynamic loads can be written as [11],

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right) = = 6h\frac{\partial U}{\partial x} - 6U\frac{\partial h}{\partial x} + 12V \tag{10}$$

where h is the fluid film thickness, μ is the absolute fluid viscosity, p represents the film pressure, U is the relative tangential velocity and V is the relative normal velocity between journal and bearing. The radial and axial directions are denoted by x and z.

The three terms on the right-hand side of Eq. (10) contribute to the hydrodynamic forces in the journal bearing. The term $6h\,\partial U/\partial x$ implies a variation of tangential velocity along the bearing surface. The second term, $6U\,\partial h/\partial x$, represents the action of the journal rotation over a wedge-shaped fluid film. Finally, the $12V$ is the expression for the radial velocity of the journal centre and is responseble for the squeeze film effect. The Reynolds' equation (10) should be expressed in term of the journal bearing variables e, ϕ and θ.

Thus, neglecting the curvature of the fluid film, the x coordinate in the unwrapped film is written as,

$$x = R\theta \tag{11}$$

and, hence,

$$\partial x = R\partial\theta \tag{12}$$

Now, using Eq. (12) together with the tangential and normal velocities given by Eqs. (6) and (8), the terms of the right-hand side of Eq. (10) can be expressed as,

$$6h\frac{\partial U}{\partial x} = 6\left(\begin{array}{l}\frac{c\dot{e}}{R}\cos\theta + \frac{ce\dot{\phi}}{R}\sin\theta + \\ +\frac{e\dot{e}}{R}\cos^2\theta + \frac{e^2\dot{\phi}}{R}\cos\theta\sin\theta\end{array}\right) \tag{13}$$

$$-6U\frac{\partial h}{\partial x} = 6\left(\begin{array}{l}\omega e\sin\theta + \frac{e\dot{e}}{R}\sin^2\theta - \\ -\frac{e^2\dot{\phi}}{R}\cos\theta\sin\theta\end{array}\right) \tag{14}$$

$$12V = 12\left(-\omega e\sin\theta + \dot{e}\cos + e\dot{\phi}\sin\theta\right) \tag{15}$$

The detailed deviation of these equations is provided in Appendix I. Using Eqs. (13) up to (15), the Reynolds' equation for finite length journal bearings is,

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right) = = 12c\left[\dot{\varepsilon}\cos\theta + \varepsilon\left(\dot{\phi} - \overline{\omega}\right)\sin\theta\right] \tag{16}$$

The Reynolds' equation given by Eq. (16) is one non-homogeneous partial differential equation of elliptical type, exact solution of which is difficult to obtain and, in general, involves a considerable numerical effort. The infinitely short [16] and infinitely long [17] journal bearing theories are commonly used to obtain approximate solutions. These two particular cases correspond to neglect the first or second term of Eq. (16).

Dubois and Ocvirk [16] considered a journal-bearing where the pressure gradient around the circumference is very small when compared with those along the length, and the first term on the left-hand side of Eq. (16) can be ignored. This assumption is, in general valid for length-to-diameter (L/D) ratios up to 0.75. Hence, the Reynolds' equation for an infinitely short journal bearing is written as,

$$\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right) = 12c\left[\dot{\varepsilon}\cos\theta + \varepsilon\left(\dot{\phi} - \overline{\omega}\right)\sin\theta\right] \tag{17}$$

The pressure boundary conditions are $p=0$ at $z=\pm L/2$ and $\partial p/\partial z = 0$ at $z=0$. Thus, applying these conditions and integrating Eq. (17) yields the pressure distribution as,

$$p = \frac{6\mu c}{h^3}\left[\dot{\varepsilon}\cos\theta + \varepsilon\left(\dot{\phi} - \overline{\omega}\right)\sin\theta\right]\left(z^2 - 0.25L^2\right) \tag{18}$$

where c is the radial clearance, θ is the angular coordinate, z is the axial direction, L represents the journal bearing length, μ is the dynamic fluid viscosity, h denotes the film thickness and ω is the relative angular velocity between the journal and bearing. The dot in the above expression denotes the time derivative of the corresponding parameter.

For an infinitely long journal bearing a constant fluid pressure and negligible leakage in the axial direction are assumed. This solution, derived by Sommerfeld [17], is valid for length-to-diameter (L/D) ratios greater than 2. Thus, ignoring the second term on the left-hand side of Eq (16) yields the Reynolds' equation for an infinitely long journal bearing as,

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) = 12c\left[\dot{\varepsilon}\cos\theta + \varepsilon\left(\dot{\phi} - \overline{\omega}\right)\sin\theta\right] \tag{19}$$

and the pressure distribution in the fluid of an infinitely long journal bearing is given by,

$$p = \frac{6\mu R^2}{c^2}\left\{\begin{array}{l}\frac{\dot{\varepsilon}}{\varepsilon}\left[\frac{1}{(1+\varepsilon\cos\theta)^2}-\frac{1}{(1+\varepsilon)^2}\right]- \\ -\frac{(\dot{\phi}-\overline{\omega})\varepsilon\sin\theta(2+\varepsilon\cos\theta)}{\left(2+\varepsilon^2\right)(1+\varepsilon\cos\theta)^2}\end{array}\right\} \qquad (20)$$

where R is the nominal journal bearing radius and the remaining parameters have the same meaning as defined previously.

Equations (18) and (20) enable the calculation of the pressure distribution in hydrodynamic infinitely short and infinitely long journal bearing as functions of the dynamic parameters and geometry of the journal bearing.

3.2. Load Carried by Journal Bearings

Once the pressure distribution is known, the load carried by the fluid film can be evaluated by integrating the pressure around the journal bearing. For any elementary segment, $Rd\theta dz$, at any angle θ to the line of centres, the fluid reaction force that equilibrates the applied load is $pRd\theta dz$. It is convenient to determine the force components in the directions tangent, F^{ε}, and perpendicular, F^{ϕ}, to the line of centres. These two force components can be evaluated as,

$$F^{\varepsilon} = -\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_{\theta_i}^{\theta_o} pR\cos\theta d\theta dz \qquad (21)$$

$$F^{\phi} = -\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_{\theta_i}^{\theta_o} pR\sin\theta d\theta dz \qquad (22)$$

The boundary conditions in the pressure distribution deal with problems related to the numerical solution of the Reynolds' equation, namely the definition of the boundary of cavitation and film regeneration. The boundary condition is a virtual line that separates the zone where there is a continuous fluid film and the zone where the film is cavitated. Thus, the force components expressed by Eqs. (21) and (22) can be obtained by integrating the pressure field either in the entire domain 2π or half domain π, corresponding to the adoption of Sommerfeld's or Gümbel's boundary conditions, respectively [18,19].

The Sommerfeld's boundary conditions, complete or full film, do not take into account the cavitation phenomenon and, consequently, contemplate the existence negative pressure for portion $\pi<\theta<2\pi$. This case is not realist in many applications due to the fluid incapacity to sustain significant sub-ambient pressures. The Gümbel's boundary conditions, which account for the rupture film, preconize the existence of a zero pressure zone for the portion between π and 2π. Therefore, the Gümbel's conditions are usually employed, being the outlet film angle θ_o evaluated as $\theta_o=\theta_i+\pi$.

4. MOBILITY METHOD

4.1. Description of the Method

The Reynolds' equation (16) in the case of finite length journal bearings does not have analytical solution, because it is an elliptical partial differential equation of second order. Therefore, the numerical methods must be used.

Broadly, the rapid methods used to perform the design of dynamically loaded journal bearings fall into three main categories, namely, the infinitely short simplification [16], the infinitely long approach [17] and methods based on the combination of these two first approaches [20]. The multigrid techniques [1] and the finite element methods [2] are also popular due their accuracy but tend to be difficult to use and require high computational effort when compared to rapid methods.

Among the different existing approaches, the mobility method developed by Booker [3,4] appears well adapted to analyze journal bearings subjected to dynamic loads in so far as it allows a quick and accurate solution. Booker [3] introduced the concept of mobility to analyze this kind of journal bearing problems. This graphical method is useful to predict the journal centre orbit marching in time from some initial eccentricity ratio on the mobility maps, as illustrated in Fig.3. However, this approach tends be tedious to use and is not adequate to incorporate in any computational program to transform it in an automatic and general method. Later Booker [4] presented a scalar form of the mobility method and extended it to numerical application. This new approach is computationally efficient because the equations of journal bearing motion are written in explicit form, avoiding the need for time consuming involved in the conventional solution of the Reynolds' equation and makes the method very fast. Based on the finite element method, Goenka [2] improved the accuracy and scope of the Booker's work to evaluate the mobility vector, maximum pressure and the angle where the maximum pressure occurs.

The mobility vector, $\vec{M}$, has the module M and is oriented, with respect to the mobility direction, by angle β, as it is illustrated in Fig.3, which allows to write $M^{\varepsilon} = M cos\beta$ and $M^{\phi} = -M\, sin\, \beta$ [13].

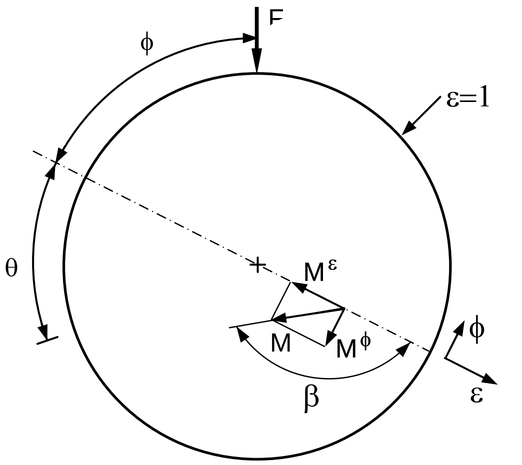

Fig.3 – Representation of mobility vector and its components.

The right hand term of Reynolds' equation (16) includes the derivative of the eccentricity ratio, $\dot{\varepsilon}$, and the time rate of the attitude angle, $\dot{\phi}$, which are the unknown quantities, and, by integration, it is possible to determine the trajectory of the journal centre. In order to simplify the solution of the Reynolds' equation, Booker [4] proposed to express the $\dot{\varepsilon}$ and $\dot{\phi}$ as function of the mobility components as,

$$\dot{\varepsilon} = \frac{F\left(\frac{c}{R}\right)^2}{\mu LD} M^{\varepsilon} \tag{23}$$

$$\dot{\phi} = \frac{F\left(\frac{c}{R}\right)^2}{\mu LD\varepsilon} M^{\phi} + \overline{\omega} \tag{24}$$

in which the mobility components M^{ε} and M^{ϕ} are given by,

$$M^{\varepsilon} = \frac{\left(I_3^{20} \cos\phi + I_3^{11} \sin\phi\right)}{\left[2\left(\frac{L}{D}\right)^2 \left(I_3^{20} I_3^{02} - I_3^{11} I_3^{11}\right)\right]} \tag{25}$$

$$M^{\phi} = -\frac{\left(I_3^{02} \sin\phi + I_3^{11} \cos\phi\right)}{\left[2\left(\frac{L}{D}\right)^2 \left(I_3^{20} I_3^{02} - I_3^{11} I_3^{11}\right)\right]} \tag{26}$$

where

$$I_n^{lm} = \int_{\theta} \frac{sin^l\, \theta\, cos^m\, \theta}{\left(1 + \varepsilon \cos\theta\right)^n} d\theta \tag{27}$$

The solution of the above equations is complex and involves a good deal of mathematical manipulation. However, Booker [4] gave the numerical solution to Eqs. (23)-(27), for positive force, *F>0,*

$$M^{\zeta} = \frac{(1-\zeta)^{\frac{5}{2}}}{\left[\pi\left(\frac{L}{D}\right)^2\right]} \tag{28}$$

$$M^{\kappa} = -\frac{4\kappa(1-\zeta)^{\frac{3}{2}}}{\left[\pi^2\left(\frac{L}{D}\right)^2\right]} \tag{29}$$

and for negative force, *F<0,*

$$M^{\zeta} = \frac{(1+\zeta)^{\frac{5}{2}}}{\left[\pi\left(\frac{L}{D}\right)^2\right]} \tag{30}$$

$$M^{\kappa} = \frac{4\kappa(1+\zeta)^{\frac{3}{2}}}{\left[\pi^2\left(\frac{L}{D}\right)^2\right]} \tag{31}$$

where

$$M^{\varepsilon} = M^{\zeta} \cos\phi + M^{\kappa} \sin\phi \tag{32}$$

$$M^{\phi} = -M^{\zeta} \sin\phi + M^{\kappa} \cos\phi \tag{33}$$

$$\zeta = \varepsilon \cos\phi \tag{34}$$

$$\kappa = \varepsilon \sin\phi \quad (35)$$

For a specified external load $F(t)$, Eqs. (23) to (35) are solved for $\dot{\varepsilon}$ and $\dot{\phi}$, and then integrated with respect to time, in order to obtain the journal centre trajectory, that is, $\varepsilon(t)$ and $\phi(t)$.

4.2. Computational Strategy

The general view of the flowchart program for analysis of journal bearings subjected to dynamic loads, based on the mobility method, is illustrated in Fig.4. The necessary steps to perform this analysis are summarized as follows,

(i) Start at instant of time t^0 with given initial conditions for eccentricity ratio ε^0, attitude angle ϕ^0 and external applied load F^0, as well as the geometric and kinematic parameters of the journal bearing L, D, c, ω, and dynamic fluid viscosity, μ;

(ii) Compute the location of the journal centre according to Eqs. (34) and (35);

(iii) Evaluate the auxiliary mobility vectors defined by Eqs. (28)-(31);

(iv) Re-evaluate the mobility components using Eqs. (32) and (33);

(v) Determine the journal velocity components, $\dot{\varepsilon}$ and $\dot{\phi}$, according to Eqs. (23) and (24);

(vi) Numerically integrate the journal velocity components in order to obtain the eccentricity ratio ε and attitude angle ϕ at the next time step. Most authors used Euler integration method rather than other more accurate method, e.g. Runge-Kutta or predictor-corrector schemes, since, for this type of problem, the computational time is substantially reduced without penalty the results accuracy;

(vii) Update the time variables, go to step (ii) and proceed with the process for the new time step. Perform these steps until the final time of analysis is reached.

5. DEMONSTRATIVE APPLICATION TO A JOURNAL BEARING SUBJECTED TO A DYNAMIC LOAD

A simple journal bearing, as illustrated in Fig.5, subjected to a dynamic load is chosen here as numerical example to demonstrate the use of the mobility method presented in the previous section. The dimensions were based on the journal bearing test rig that has been studied by Bouldin and Berker [21], which, in turn, was chosen to be representative of a common journal bearings used in automobile engines. Journal bearings of slider-crank mechanism of internal combustion engines are typical examples of dynamically loaded journal bearings that present complex load diagram as well as unsteady relative motion.

The journal bearing properties and initial conditions are listed in Table 1. The length-to-diameter (L/D) of this journal bearing is 0.4, and, therefore, it can be considered a short journal bearing. The full time simulation corresponds to two full crank rotations. The dynamic fluid viscosity is taken as to 4.16 cP, which is a typical value used in internal combustion engines. The external load applied on the journal bearing is taken from reference [21] presents the form of Fig. 6. The load is plotted against the crank angle.

In the present example, the value for journal centre motion and the attitude angle are plotted as a quantitative measure of the performance of the journal bearing, as shown in Figs. 7a and 7b. In addition, the maximum pressure distribution at the central plane of the journal bearing and minimum fluid thickness are also presented and analyzed, as illustrated in Figs. 8 and 9, respectively.

Figure 7a shows the journal centre trajectory for the initial conditions for eccentricity ratio and attitude angle equal to ε^0=0.1 and ϕ^0=1.5 rad, respectively. It is observable that the external load produces an unsteady motion of the journal within

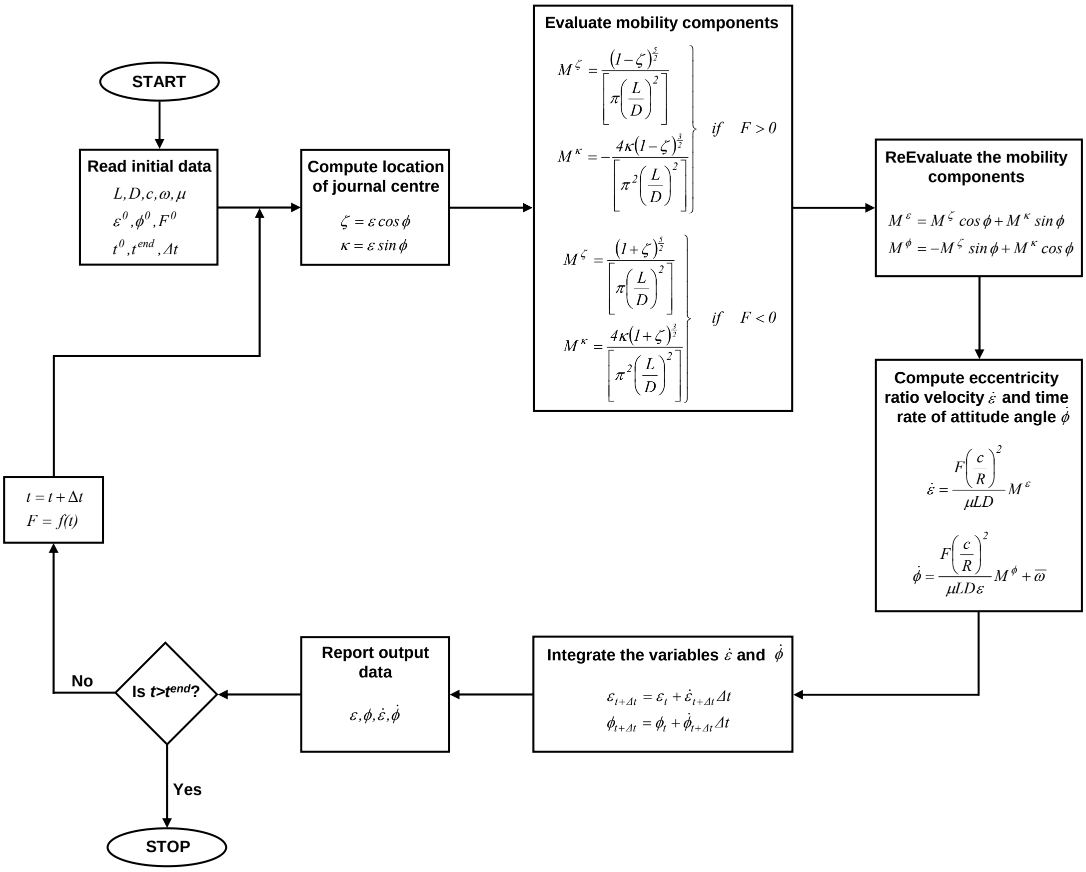

Fig.4 – Computational flowchart of the mobility method.

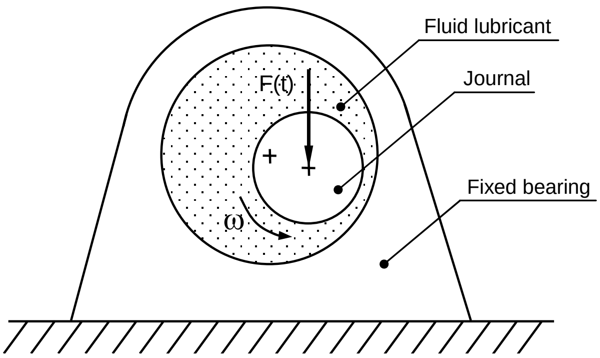

Fig.5 – Journal bearing subjected to a variable external load.

Tab.1 – Journal bearing properties.

Journal bearing length	25.40 mm	Initial eccentricity, ε^0	0.1
Journal bearing diameter	63.50 mm	Initial attitude angle, ϕ^0	1.5 rad
Radial clearance	35.56 mm	Initial time simulation	0.0000 s
Dynamic fluid viscosity	4.16 cP	Final time simulation	0.0060 s
Angular velocity	2000 rpm	Integration time step	0.0001 s

the bearing boundaries. It should be noted that with this approach of mobility method the choice of the initial conditions does not affect the relative journal bearing motion, only influences the first instants of simulation. Thus, the cyclic behaviour of the journal, independent of the assumed initial values for ε^0 and ϕ^0, is reached very quickly. In Fig.7b the journal bearing attitude angle is shown as function of the crank angle. In dynamically loaded journal bearings both the eccentricity and attitude angle vary with the cycle of applied load, and the correct design should ensure that the combination of load and speed rotation does not lead a dangerous small minimum film thickness.

Since the length-to-diameter ratio of the journal bearing is equal to 0.4, the pressure distribution can be evaluated using Eq. (18) in which the positive pressure region lies between θ_i and θ_o obtaining by equating Eq. (18), yielding,

$$\theta_i = tan^{-1}\left(-\frac{\dot{\varepsilon}}{\varepsilon(\dot{\phi}-\overline{\omega})}\right) \quad (36)$$

$$\theta_o = \theta_i + \pi \quad (37)$$

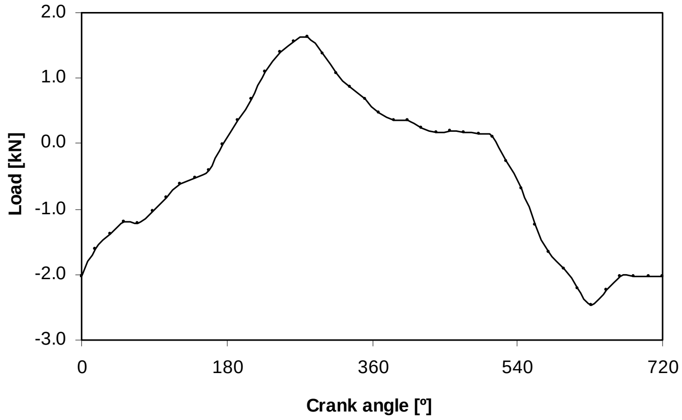

Fig.6 – Real journal bearing load presented by Bouldin and Berker [21].

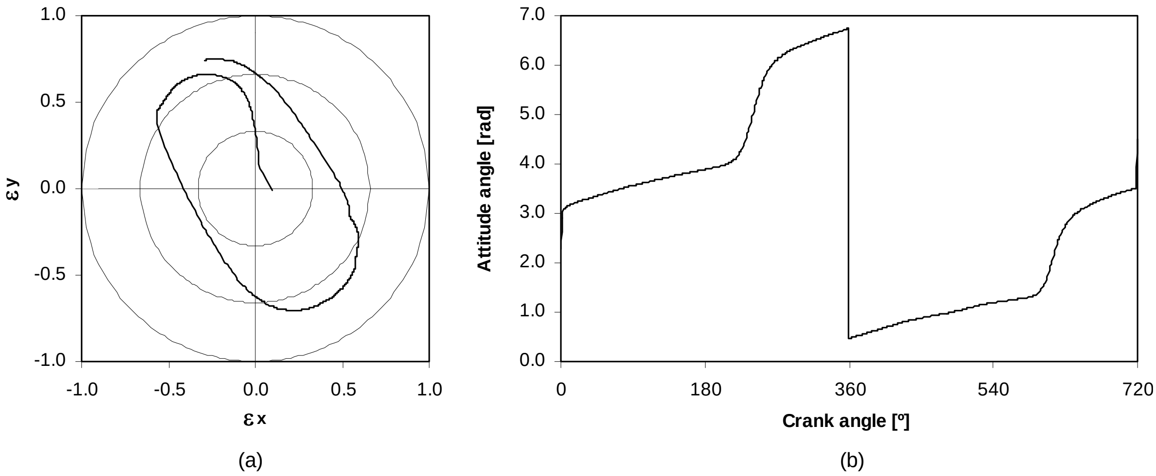

Fig.7 – (a) Journal centre motion; (b) Attitude angle as function of the crank angle.

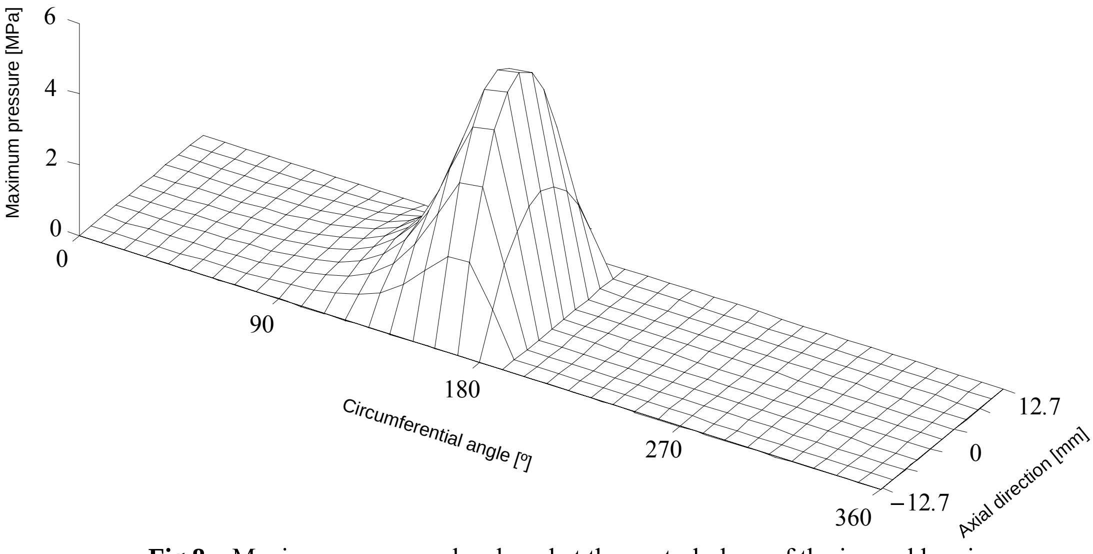

Fig.8 – Maximum pressure developed at the central plane of the journal bearing.

It is important to note that angles θ_i and θ_o change with the work conditions of the journal bearing. Thus, for each instant of simulation, the pressure is function of the journal bearing geometric properties, its kinematic characteristics and varies in both circumferential and axial directions.

The maximum pressure field occurs at the central plane of the journal bearing, being the pressure profile plotted in Fig.8. The region of the positive pressure is limited to half of the circumferential extension, and in the other half the film is assumed to rupture due to the existence of sub-ambient pressures, as Gümbel's boundaries conditions preconize.

The minimum film thickness is related to the eccentricity ratio and radial clearance as follows,

$$h_{min} = c(1-\varepsilon) \quad (38)$$

The variation of the minimum film thickness during the dynamic simulation of journal bearing gives the value of the absolute minimum film thickness. This value should be greater than the average asperity height of the journal bearing surfaces in order to prevent the metal-to-metal contact and, consequently, to avoid the friction and wear, besides keeping the hydrodynamic approach valid. From the journal bearing design view point, the safe value for film thickness is 2.5 μm, and, in practical, the minimum film thickness should be at least (1.0-1.5)×2.5 μm [15]. Figure 9 shows the variation of the minimum film thickness with the crank angle. The dashed horizontal line in the same figure represents the safe film thickness. By observing Fig.9, it clear that the effective hydrodynamic lubrication is performed on the journal bearing, meaning that the journal and bearing surfaces are completely separated.

6. CONCLUDING REMARKS

In this work, the analytical mobility method for dynamically loaded journal bearings was presented, with the intent to include it in a general computational program, such as the DAP (Dynamic Analysis Program), that has been developed for the dynamic analysis of general mechanical systems. An illustrative example and numerical results were presented, being the efficiency of the method discussed in the process of their presentation.

The mobility method seems to be quite useful and numerically efficient when compared to others approaches such as the multigrid techniques based on the Elrod algorithm and the finite element methods of analysis. In fact, the mobility method is sufficiently accurate, fast and easy to understand and apply by designers and engineers that do not need to be very expertise in the field of tribology.

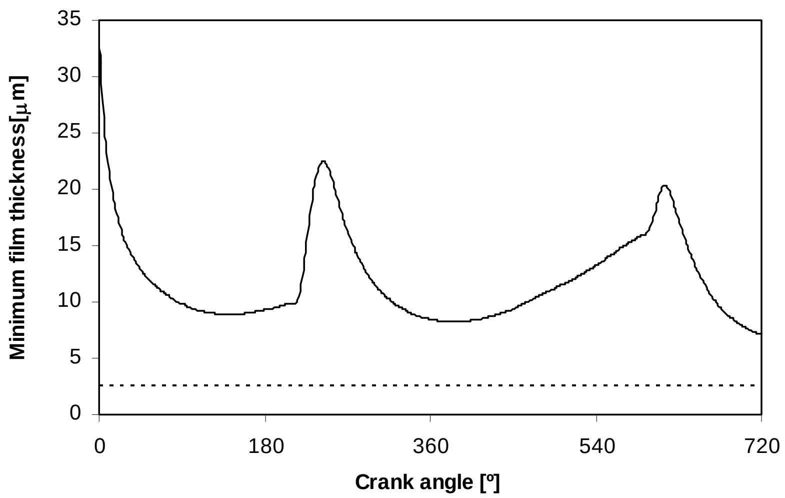

Fig.9 – Variation of the minimum fluid film thickness.

In general, in the dynamically loaded journal bearings the classic analysis problem is predicting the motion of the journal centre under arbitrary and known loading, such as in the example presented in this work. Conversely, when simulating mechanism with journal bearings, the time variable parameters are known from the dynamic analysis and the instantaneous forces on the journal bearings are evaluated. Thus, in order to incorporate the mobility method in a computational program for dynamics of mechanisms, the mobility method has to be inverted.

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NOMENCLATURE

c radial clearance between the journal and bearing, $c=R_B-R_J$, [L]

D nominal diameter of the journal bearing, [L]

e eccentricity [L]

$\dot{e}$ variation or radial velocity [LT^{-1}]

F journal bearing applied load [F]

h fluid film thickness [L]

L journal bearing length [L]

M mobility vector

p hydrodynamic film pressure [FL^{-2}]

R nominal journal bearing radius [L]

t time [T^{-1}]

U tangential velocity component [LT^{-1}]

V normal velocity component [LT^{-1}]

ε eccentricity ratio, $\varepsilon=e/c$, [-]

$\dot{\varepsilon}$ non-dimensional eccentricity ratio variation, $\dot{\varepsilon} = \dot{e}/c$, [T^{-1}]

θ coordinate in the circumferential direction [-]

θ_i film inlet edge position [-]

θ_o film outle edge position [-]

μ absolute viscosity of fluid lubricant [FL^{-2}T]

ϕ attitude angle [-]

$\dot{\phi}$ time rate of attitude angle [T^{-1}]

ω relative angular velocity [T^{-1}]

$\overline{\omega}$ average angular velocity, $\overline{\omega} = \omega/2$, [T^{-1}]

APPENDIX I

The x coordinate in the unwrapped film is,

$$x = R\theta \tag{A1}$$

and, consequently,

$$\partial x = R\partial\theta \tag{A2}$$

The tangential and normal velocities of the journal bearing are expressed as,

$$U = \omega R + \dot{e}\sin\theta - e\dot{\phi}\cos\theta \tag{A3}$$

and,

$$V = \omega\frac{\partial h}{\partial\theta} + \dot{e}\cos\theta + e\dot{\phi}\sin\theta \tag{A4}$$

Thus, using Eqs. (A2) and (A3), $\partial U/\partial x$ can be obtained as,

$$\frac{\partial U}{\partial x} = \frac{1}{R}\frac{\partial U}{\partial\theta} \tag{A5}$$

$$\frac{\partial U}{\partial x} = \frac{1}{R}\frac{\partial}{\partial\theta}\left(\omega R + \dot{e}\sin\theta - e\dot{\phi}\cos\theta\right) \tag{A6}$$

$$\frac{\partial U}{\partial x} = \frac{1}{R}\left(\dot{e}\cos\theta + e\dot{\phi}\sin\theta\right) \tag{A7}$$

Since, the fluid film thickness is expressed by,

$$h = c + e\cos\theta \tag{A8}$$

and therefore,

$$6h\frac{\partial U}{\partial x}=6(c+e\cos\theta)\frac{1}{R}\left(\dot{e}\cos\theta+e\dot{\phi}\sin\theta\right) \tag{A9}$$

$$6h\frac{\partial U}{\partial x}=6\left(\frac{c\dot{e}}{R}\cos\theta+\frac{ce\dot{\phi}}{R}\sin\theta+\frac{e\dot{e}}{R}\cos^2\theta+\frac{e^2\dot{\phi}}{R}\cos\theta\sin\theta\right) \tag{A10}$$

From Eqs. (A2) and (A8), $\partial h/\partial x$ can be written as,

$$\frac{\partial h}{\partial x}=\frac{1}{R}\frac{\partial h}{\partial\theta} \tag{A11}$$

$$\frac{\partial h}{\partial x}=\frac{1}{R}\frac{\partial}{\partial\theta}(c+e\cos\theta) \tag{A12}$$

$$\frac{\partial h}{\partial x}=\frac{1}{R}(-e\sin\theta) \tag{A13}$$

and, consequently, the term $-6U\partial h/\partial x$ is expressed as follows,

$$-6U\frac{\partial h}{\partial x}=-6\left(\omega R+\dot{e}\sin\theta-e\dot{\phi}\cos\theta\right)\frac{1}{R}(-e\sin\theta) \tag{A14}$$

$$-6U\frac{\partial h}{\partial x}=6\left(\omega e\sin\theta+\frac{e\dot{e}}{R}\sin^2\theta-\frac{e^2\dot{\phi}}{R}\cos\theta\sin\theta\right) \tag{A15}$$

From Eq. (A8), $\partial h/\partial\theta$ can be written as,

$$\frac{\partial h}{\partial\theta}=\frac{\partial}{\partial\theta}(c+e\cos\theta) \tag{A16}$$

$$\frac{\partial h}{\partial\theta}=-e\sin\theta \tag{A17}$$

Then, the term $12V$ is obtained as,

$$12V=12\left[\omega(-e\sin\theta)+\dot{e}\cos\theta+e\dot{\phi}\sin\theta\right] \tag{A18}$$

$$12V=12\left(-\omega e\sin\theta+\dot{e}\cos\theta+e\dot{\phi}\sin\theta\right) \tag{A19}$$

Combining Eqs. (A10), (A15) and (A19), the right-hand side of the Reynolds' equation (10) is given by,

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right)=6c\left(\frac{\dot{e}}{R}\cos\theta+\frac{e\dot{\phi}}{R}\sin\theta+\frac{2e\dot{e}}{cR}-\frac{\omega e}{c}\sin\theta+\frac{2\dot{e}}{c}\cos\theta+\frac{2e\dot{\phi}}{c}\sin\theta\right) \tag{A20}$$

Since $\varepsilon=e/c$ and $\dot{\varepsilon}=\dot{e}/c$, Eq. (A20) can be rewritten as,

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right)=6c\left(\frac{\dot{\varepsilon}c}{R}\cos\theta+\frac{\varepsilon c\dot{\phi}}{R}\sin\theta+\frac{2c^2\varepsilon\dot{\varepsilon}}{cR}-\omega\varepsilon\sin\theta+2\dot{\varepsilon}\cos\theta+2\varepsilon\dot{\phi}\sin\theta\right) \tag{A21}$$

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right)=6c\left[\frac{c}{R}\left(\dot{\varepsilon}\cos\theta+\varepsilon\dot{\phi}\sin\theta+2c\varepsilon\dot{\varepsilon}\right)+\varepsilon\left(2\dot{\phi}\sin\theta-\omega\sin\theta\right)+2\dot{\varepsilon}\cos\theta\right] \tag{A22}$$

For typical journal bearings, $c/R \ll 1$, then the final form of Reynolds' equation is given by,

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right)=12c\left[\dot{\varepsilon}\cos\theta+\varepsilon\left(\dot{\phi}-\overline{\omega}\right)\sin\theta\right] \tag{A23}$$

where $\overline{\omega}=\omega/2$ is the average angular velocity of the journal and bearing.