THE EFFECT OF STIFFENENG RINGS ON THE DEFLECTION OF CIRCULAR ISOTROPIC PLATES UNDER STATIC LOADING

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ABSTRACT

This paper presents an analysis of the deflection of isotropic circular plates with stiffening rings under the action of static pressure. The linear equation with variable coefficients governing the deflection of circular plates under such loading is derived. The effect of the stiffening rings are introduced by approximating the varying thickness/ plate radius by a Fourier series. Numerical examples are illustrated for non-stiffened plates of two thicknesses and two stiffened plates, with one and two stiffening rings respectively, all under clamped-edge boundary condition. Results showed the role of the stiffening rings in minimizing the deflection of the plate and eventually the induced stresses under the action of external pressure. Moreover, the present model results showed close agreement with those obtained by applying a finite element analysis using ANSYS code consequently, illustrating the versatility of this simple and elegant method.

Keywords: *Industrial Application, Plates, Pressure Sensors*

Notations

$W(r,\theta)$	Transverse deflection
r,θ	Polar coordinates of a point in the mid-plane of plate
a	Outer radius of plate
y	Plate thickness as a function of radius
h1	Minimum plate thickness
h2	Plate with stiffening rib thickness
ν	Poisson's ratio
E	Young modulus
$\phi(r,\theta)$	Slope of deflected plate
ρ	Plate material density
x	Non dimensional radius parameter = r / a

INTRODUCTION

In general engineering applications, plates and shell type structures are frequently used (e.g. wing panels, ribs, fuselage, aircraft cockpits; car bodies). Most of these structures are made of light materials and "stiffened" using longitudinal and transverse members (spars, ribs ,etc.)[1].

Flat circular plates are widely used in electrical transducers either by sensing the center deflection with some displacement transducer or by adhering strain gages to the plate surface [2]. Most recent applications of micro-pressure sensors innovated the implantation of piezoresistors beneath the surface of a silicon die acting as a miniaturized pressure sensing plate or diaphragm [3]. The full scale deflection at the center of the plate (or diaphragm) must be about one third of its thickness to keep non-linearity less than 5%. However, if local strains rather than center deflections (as is the case of micro-pressure sensors) is measured and in order to keep linearity, stiffened plates or diaphragms are often needed to cover a pressure range from 0.5 in H2O up to 500 Pa.[4]. Although the stiffened rectangular plates has been thoroughly studied [5], the application of stiffeners to circular plates is not as much popular.

The objective of the current study is to include the effect of the stiffening rings to the solution of the linear differential equation with variable coefficients governing the deflection of circular plates under static pressure loading by adopting a varying thickness function. The thickness is simulated as a radial distance repeated function approximated by a Fourier Series. The numerical results obtained by applying this procedure are compared to results obtained by solving the same problem by applying finite element method using *ANSYS* code.

PROBLEM FORMULATION

The linear differential equation with variable coefficients governing the deflection of circular plates under external pressure is given as [6],

$$\frac{d^2\phi}{dx^2} + \left(\frac{1}{x} + \frac{d\ln y^3}{dx}\right)\frac{d\phi}{dx} - \left(\frac{1}{x^2} - \frac{\nu}{x}\frac{d\ln y^3}{dx}\right)\phi = -\frac{px}{y^3} \qquad (1)$$

where symbols are as appearing in notation and p and y are;

$$p = \frac{6\left(1-\nu^2\right)\cdot a^2 \cdot q}{E \cdot h^3} \qquad (2)$$

Assuming that the stiffening rings are of equal dimensions and are equally spaced along the radius of the plate, the thickness of a stiffened plate may be then looked after as a spatial domain periodic function. A periodic function is that function that repeats itself over and over again. Furthermore, as this function meets the *Dirchlet* conditions as it is single valued, finite, and have a finite number of discontinuities and maxima and minima in one cycle, it may be represented by a Fourier series. That is

$$y = \sum_{k=0}^{10}\left(a_k \cos(kx) + b_k \sin(kx)\right) \qquad (3)$$

where

$$a_0 = \frac{1}{2T}\int_0^{2T} f(x)dx$$

$$a_k = \frac{1}{T}\int_0^{2T} f(x)\cos(kx)\,dx$$

$$b_k = \frac{1}{T}\int_0^{2T} f(x)\sin(kx)\,dx$$

NUMERICAL EXAMPLE

In order to illustrate the potential of this method the following example is to be considered for analysis. A circular copper plate (or diaphragm) clamped at the edge and closed at the center with the dimensions and material properties as appearing in table 1.

The Fourier series defining the stiffened plate or diaphragm thickness, assuming very sharp changes in thickness, is truncated up to 10 harmonics. The two and the one stiffened plates or diaphragms differed in the width span of the (cyclic period (T)) as appearing in figure(1a), (1b).

Introducing the Fourier series representation for the thickness function into equation(1), calculation for the slope was carried out numerically using MATHCAD code considering zero slope conditions at the boundaries r=0 & r=a.

Results were then integrated to obtain the deflection overall the considered margin. Furthermore, the same cases were checked by solving the problem using the finite element technique. *ANSYS* code was used to calculate the deflection of the said examples using (elastic shell element 93) element type and (1200 nodes).

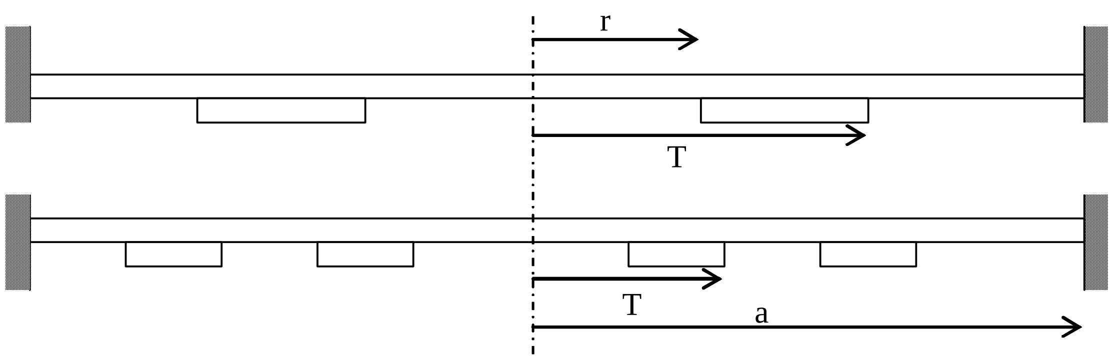

Fig 1 - a) One Stiffener Diaphragm b) Two Stiffener Diaphragm

Table 1 - Dimensions and material properties for the examples considered in this paper

	Case(1)	*Case(2)*	*Case(3)*	*Case(4)*
h1,Minimum thickness mm.	0.1	0.1	0.1	-
h2,Maximum thickness mm.	-	0.4	0.4	0.4
a, Outer radius of plate mm.	41	41	41	41
Cyclic period (T) mm	-	(4/5) .a	(2/5).a	-
..Poisson's ratio	0.327	0.327	0.327	0.327
E , Young modulus GPa	11.9	11.9	11.9	11.9

RESULTS AND DISCUSSION

As pointed out earlier, all the computation were carried out using either the MATHCAD code or the ANSYS finite element code for the examples illustrated in table (1).

Figure (2) shows the deflection of the four different cases tabulated in table (1) against radial coordinates obtained by using the analytical method and assuming a uniform distributed pressure of (10 KPa) on surface. All the four cases show the same trend as the maximum deflection occurs at the center (r=0) of the diaphragm. However, the results also clearly indicate the role of the stiffening rings in minimizing the influence of the outside pressure on the deflection of the diaphragms both in magnitude and shape. Apparently introducing only one stiffening ring would reduce the maximum deflection occurring at the diaphragm mid span by about 300%. Yet, adding a second stiffening ring would only decrease the mid span deflection by about 75%, otherwise, the deflection at distant peripheral points is rarely affected by the presence of the second stiffening ring. The deflected form of the two stiffened ring shows also a tendency of flattening at the points near the mid span unlike the trend of the deflected curves of the non stiffened and the one-ring stiffened diaphragms.

Figures 3 & 4 show a comparison of the results obtained by applying the recent thickness approximation technique and those obtained by the finite element method for cases 2 and 3 respectively. Obviously, both figures indicate very good agreement of the results of the two methods for the entire spatial region with maximum difference occurring at the mid span region. The tendency of the flattening near the mid span for the two rings stiffened diaphragm, case 3, is emphasized in figure 4. This trend is due to the presence of the two rings with the inner ring closer to the middle causing eventually the decrease in deflection at that area, keeping in mind that the decrease in deflection means the decrease in the stress fields at that region, consequently the diaphragm is usable for higher pressure values with the same basic dimensions.

Finally, figure 5 demonstrates the relation of the external applied uniform pressure versus the maximum deflection at the center point of the diaphragm for all of the studied cases. All the cases showed that the maximum deflection rises linearly with the increasing pressure. However, the slope of the linear deflection / pressure relationship differs according to the thickness and to the presence and number of the stiffening rings as the lowest thickness case with no stiffening rings showed the higher slope relationship. Introducing one stiffening ring reduced the slope nearly three folds, yet, if another stiffening ring is added the slope only decreased about 75%.

CONCLUSIONS

An analysis of the deflection of isotropic circular plates with stiffening rings under the action of static pressure has been studied by solving the linear equation with variable coefficients governing the deflection of circular plates under such loading and introducing the effect of the stiffening rings by approximating the varying thickness/ plate radius by a Fourier series.

1- It was found that introducing a stiffening ring would cause lower maximum deflection at the center of the diaphragm. If two stiffening rings are added, the deflection at the center is still lowered with a flattened deflected zone at the mid-span of the diaphragm.

2- The results obtained by applying the new method showed very good agreement to those obtained by applying the finite element method.

3- Pressure to deflection linearity was seen to be conserved for all the studied cases. However the adding of stiffening rings would lower the slope of the linear relationship.

4- One stiffening ring is seen to offer the best increase in(pressure range) to the decrease in (sensitivity) compromise for pressure sensing diaphragms.

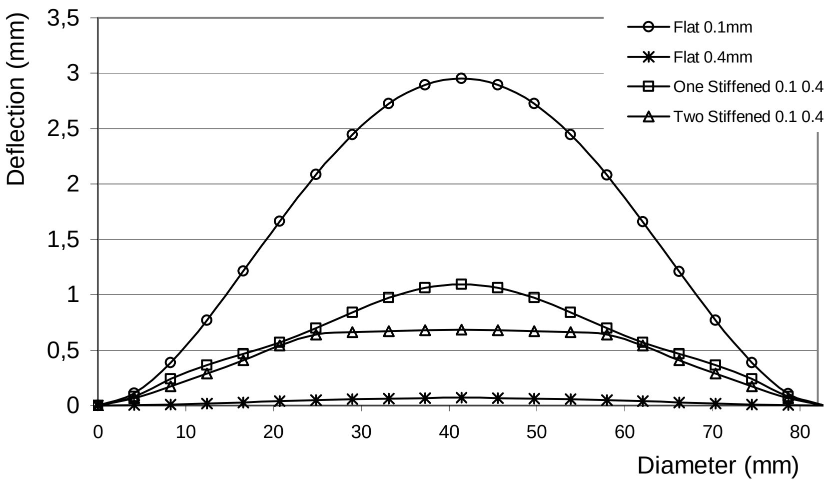

Fig 2 - Deflection of the four different cases against radial coordinates obtained by using the analytical method

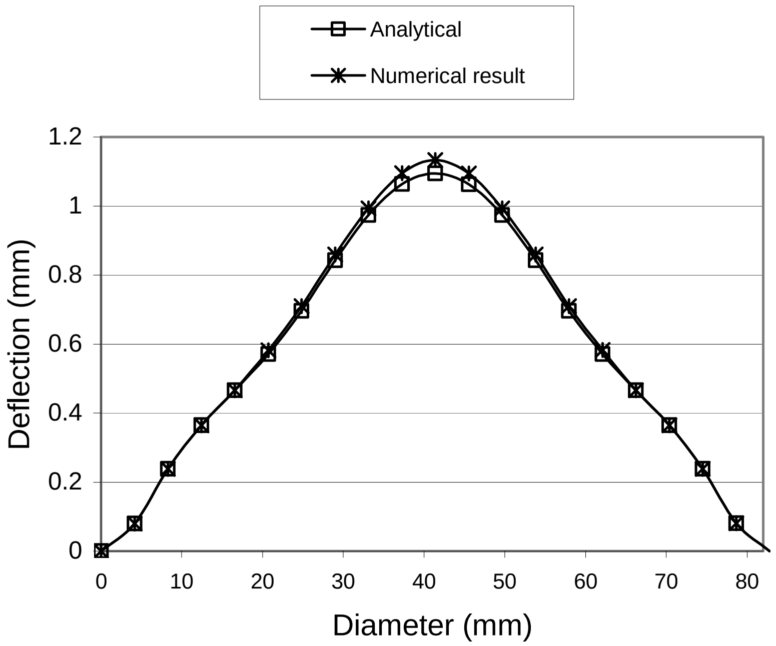

Fig 3 - Comparison of the results obtained by applying the recent thickness approximation technique and those obtained by the finite element method for case 2

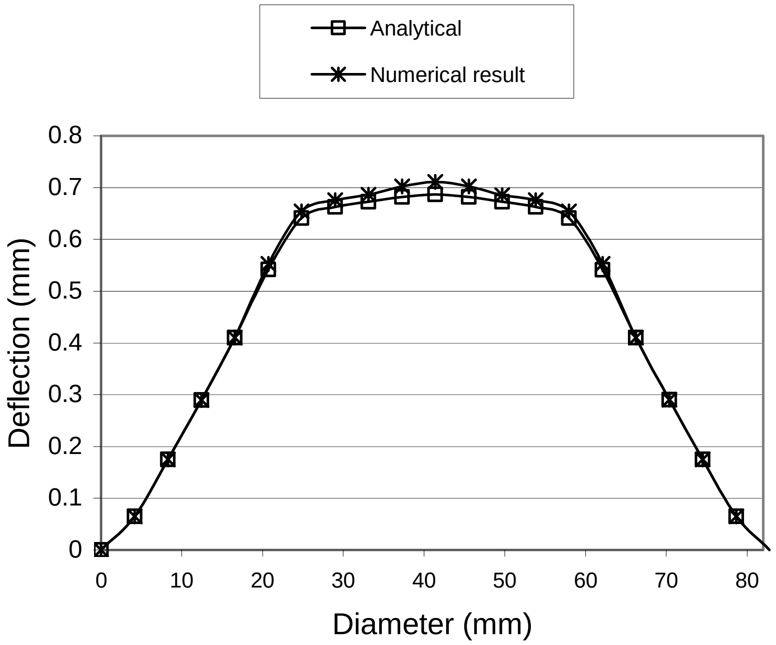

Fig 4 - -Comparison of the results obtained by applying the recent thickness approximation technique and those obtained by the finite element method for case 3

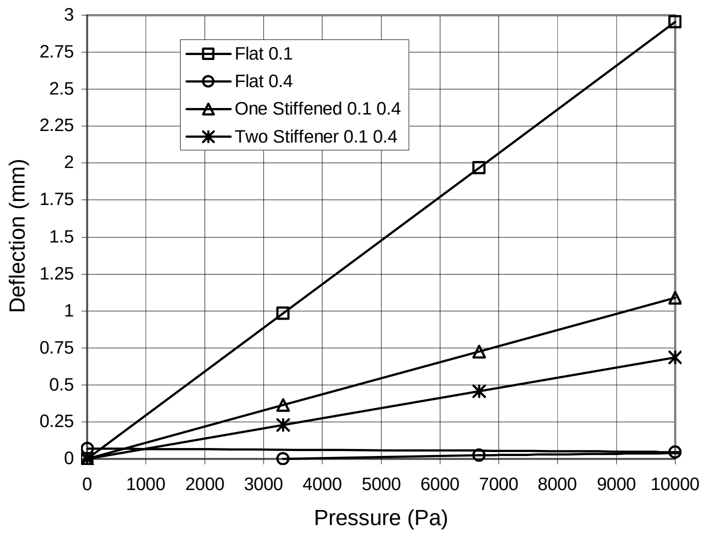

Fig 5 - The external applied uniform pressure versus the maximum deflection at the center point of the diaphragm for all of the studied cases

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